Trade, Technology and the Great Divergence*

(or An Economic History of the World in Just Thirty-Six Equations)

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Abstract

This paper develops a model that captures the key features of the Industrial Revolution and the Great Divergence between the industrializing "North" and the lagging "South." In particular, a convincing story is needed for why North-South divergence occurred so dramatically during the late 19th Century, a good hundred years after the beginnings of the Industrial Revolution. To this end we construct a trade/growth model that includes both endogenous biased technologies and intercontinental trade. The Industrial Revolution began as a sequence of unskilled-labor intensive innovations which initially incited fertility increases and limited human capital formation in both the North and the South. The subsequent co-evolution of trade and technological growth however fostered an inevitable divergence in living standards - the South increasingly specialized in production that worsened their terms of trade and spurred even greater fertility increases and educational declines. Biased technological changes in both regions only reinforced this pattern. The model highlights how pronounced divergence ultimately arose from interactions between specialization from trade and technological forces.

- *Keywords*: Industrial Revolution, unified growth theory, endogenous growth, demography, skill premium, Great Divergence
- *JEL Codes*: O, F, N

*Preliminary draft. Please do not quote.

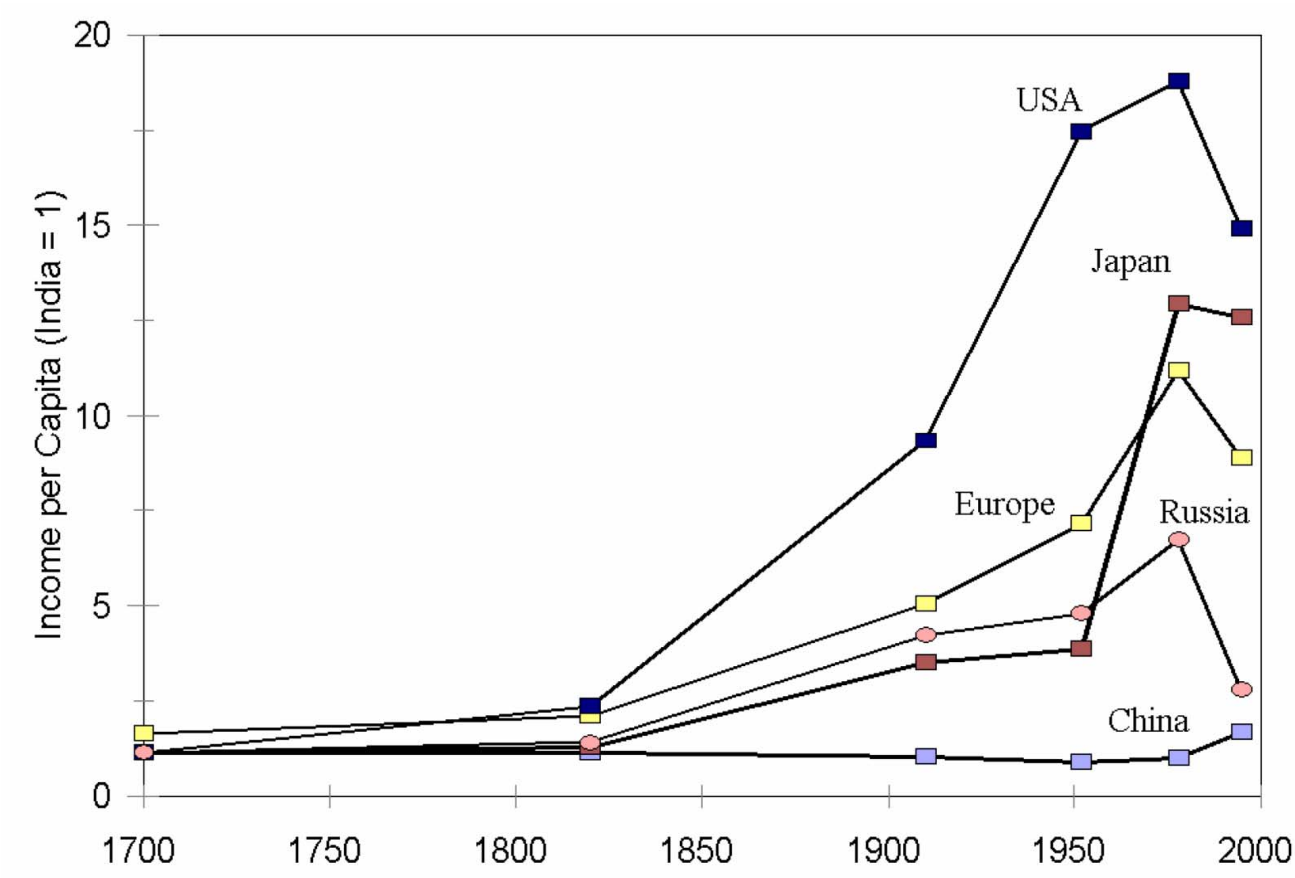
1 Introduction

The last two centuries have witnessed dramatic changes in the global distribution of income and population. At the dawn of the Industrial Revolution, living standards between the richest and poorest economies of the world were roughly 2 to 1. With industrialization came both income and population growth within a few core countries. But massive divergence in living standards across the globe did not take place until the latter half of the 19th century, the time when the first great era of globalization started to take shape (see Figure 1). Today the gap between material living standards in the richest and poorest economies of the world is 30 or 40 to 1, in large part due to the events of the 19th century. It seems an interesting coincidence then that such unprecedented growth in inter-continental commerce (conceivably creating a great force for convergence by exploiting comparative advantages and facilitating flows of knowledge) coincided so precisely with unprecedented divergence in living standards across the world. Why did incomes diverge just as the world became flat? These phenomena beg for an explanation. This paper argues that trade and technological growth patterns *together* sowed the seeds for divergence, contributing enormously to today's great wealth disparity.

Some important "stylized facts" from economic history motivate our theory. One concerns the nature of industrialization itself - technological change was unskilled labor-intensive during the early Industrial Revolution but became relatively skill-intensive during the latter nineteenth century. Indeed, England's early industrialization was in many respects largely a revolution in the cotton textile industry, with the adoption of the factory system of production and its associated new machinery. The textile industry was revolutionary in its ability to employ large numbers of unskilled and uneducated workers with minimal supervision, thus diminishing the productive role for skilled labor and education (Galor 2005; Clark 2007). By the 1850's, however, two major changes in technological growth occurred - it became much more widespread, and it became far more complementary to skilled workers (Mokyr 2002).

Another historical feature of great importance was the rising role of international trade in the world economy. Inter-continental commerce between "western" economies and the rest of the world (what we might mildly mislabel as "North-South" trade) was not particularly robust until the latter half of the 19th century. By the 1840s steam ships were faster and more reliable than sailing ships, but their high coal consumption limited how much cargo they could transport; consequently only very light and valuable freight was shipped (O'Rourke and Williamson 1999). But by 1870 a number of innovations dramatically reduced the cost of steam ocean transport, and real ocean freight rates fell by nearly 35% from 1870 to 1910 (Clark and Feenstra 2003). By 1900 the economic centers of the "South" such as Alexandria, Bombay and Shanghai were fully integrated into the British economy, both in terms of transport costs and capital markets (Clark 2007). Thus, while a closed economy model would be more appropriate to describe the first stages of the Industrial Revolution (1750-1850), a more open economy framework would better

Figure 1: Incomes per Capita Relative to India



sources: Maddison (1989), Prados de la Escosura (2000), Penn World Tables describe the latter stages of industrialization (1850-1910).

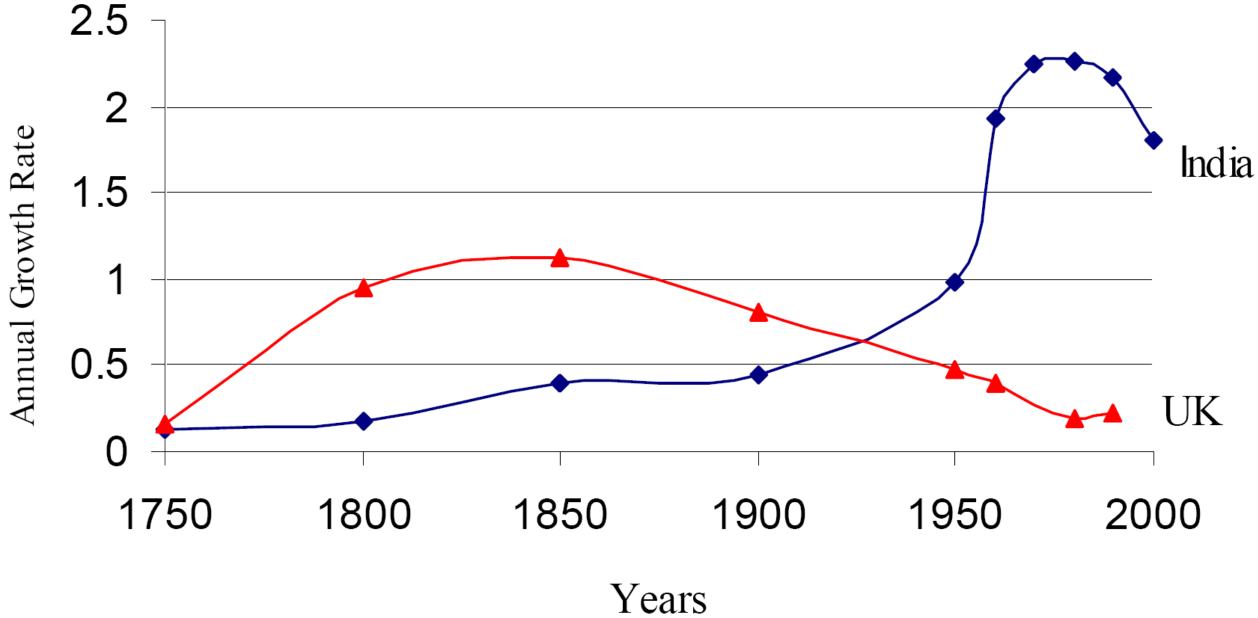
To analyze the intellectual puzzle of the Great Divergence, we develop a model that has a number of key features which mimic these historical realities. The first feature of our approach is that we endogenize the direction and extent of technological change in both regions. Technologies are sector specific, and sectors have different degrees of skill intensities. Following the endogenous growth literature, we allow potential innovators to observe the employment of factors in different sectors, and tailor their research efforts towards particular sectors. Thus the scope and direction of innovation will depend on each region's employment and demography.

The second key feature is that we endogenize demography itself. More specifically, we allow households to make education and fertility decisions based on market wages for skilled and unskilled labor. The method is similar to other endogenous demography models where households face a quality/quantity tradeoff with respect to their children. Thus, when the premium for skilled labor rises families choose to have fewer but better educated children.

The final feature is that we allow for burgeoning trade between the North and the South. During the initial stages of industrialization, trade is not possible due to prohibitively high transport costs. These costs however exogenously decrease over time; at a certain point trade becomes feasible, at which time the South exchanges labor-intensive products for the North's skill-intensive products. At this stage development paths begin to diverge - the North's specialization in skilled production produces a demographic transition, while the South's specialization in unskilled production generates unskilled-labor intensive technological growth but consequently produces more population as well.

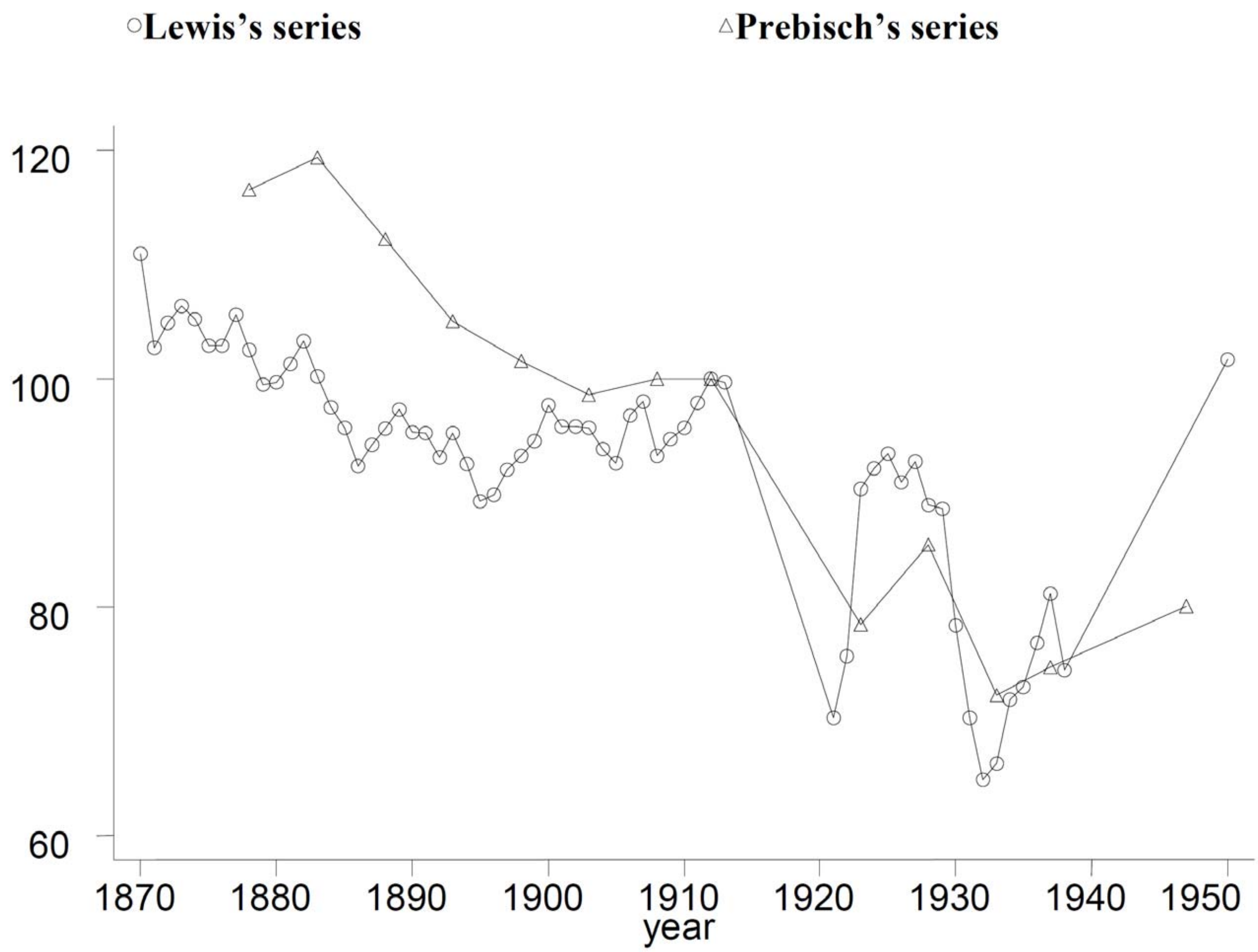
With this basic setup, we simulate the model to roughly capture the main features of industrialization and divergence between the North and the South from roughly 1700 to 1900. Because of the great abundance of unskilled labor in the world, innovators first develop unskilled-intensive technologies. Thus early industrialization is characterized by unskilled intensive technological growth and population growth *both* in the North and the South; consequently living standards in the two regions do not diverge during this time. Once trade becomes possible, however, the North starts specializing in skill-intensive innovation and production. This induces a demographic transition of falling fertility and rising education rates in the North. The South of the other hand specializes in unskilled-intensive production, inducing both unskilled-intensive technological growth and further population growth (see Figure 2). This population divergence actually fosters deterioration in the South's terms of trade, forcing the South to produce more and more primary commodities for skill-intensive products and generating even more fertility increases (see Figure 3). Thus the South's static gains from trade become a dynamic impetus to prosperity, and living standards between the two regions diverge dramatically as a result.

Figure 2: Population Growth Rates in the U.K. and India



source: Maddison (2001)

Figure 3: Relative Price of Primary Products According to Lewis and Prebisch 1870-1950 (1912 = 100)



source: Hadass and Williamson (2003)

Alternative Stories of Divergence

We argue that analyzing the interactions between the North and the South, and between trade and technological flows, is critical to understanding both the Industrial Revolution and the Great Divergence. In contrast many explanations of divergence rely on institutional differences between regions of the world (North and Thomas 1973, Acemoglu et al. 2001, 2005). From this perspective economic growth is a matter of establishing the right "rules of the game," and underdevelopment is simply a function of some form of institutional pathology. In the end however it is unclear if institutions made any difference to the course of European growth, or if inefficient institutions had any quantifiable and meaningful affect on growth in peripheral regions (Clark 2007). For example, Pomeranz (2000) argues that by 1800 China had an economic system that was as developed, market driven and individually rational as Europe's. The same has been said of India, whose institutions of secure property rights, free trade, fixed exchange rates and open capital markets were nearly "ideal" for development (Clark and Wolcott 2003).

Another potential explanation for the divergence is that peripheral countries were specializing in *inherently* less-productive industries (Galor and Mountford 2006, 2008). But this also is not very convincing - so called low-technology sectors such as agriculture enjoyed large productivity advances during the early stages of the Industrial Revolution (Bekar and Lipsey 1997, Clark 2007). And in the twentieth century, developing countries specialized in textile production which of course experienced massive technological improvements more than a century before.

Another related puzzle is the scale of the developing world. If a full one third of the world had become either Indian or Chinese by the twentieth century (Galor and Mountford 2002), why were Indians and Chinese not more wealthy? After all, most semi-endogenous and endogenous growth theories have some form of scale effect, whereby large populations can create a boon to innovative activities (Acemoglu, forthcoming).[1] Any divergence story that focuses on the explosive population expansion in peripheral economies faces this awkward implication from the canonical growth literature.

Relation to Galor and Mountford's "Trade and the Great Divergence"

The paper presented here relates most closely and obviously to Oded Galor and Andrew Mountford's theoretical works on the Great Divergence (Galor and Mountford 2006, 2008) (henceforward 'GM'). These papers similarly suggest that the South's specialization in unskilled-intensive production stimulated fertility increases which lowered per capita living standards. However, we offer a unique narrative in describing both the North's launch into modernity and

[1] More specifically, in such seminal endogenous growth models as Romer (1986, 1990), Segerstrom, Anant and Dinopolous (1990), Aghion and Howitt (1992), and Grossman and Helpman (1991), a larger labor force implies faster growth of technology. In "semi-endogenous" growth models such as Jones (1995), Young (1998), and Howitt (1999), a larger labor force implies a higher level of technology.

the South's vicious cycle of underdevelopment that is quite distinct in a number of ways.

The first involves the nature of trade - since we are concerned with an era with limited exchanges of differentiated products and little intra-industry trade, trade is best modeled as Ricardian (based on productivity differences) or Hecksher-Ohlin oriented (based on factor differences).[2] GM uses the former approach, while we use both. Such a hybrid trade model seems most appropriate to us given the rather large differences in factor technologies and skewed factor endowments between the North and the South for the period we study.

Second, we endogenize both the scope and the direction of technological progress in both regions. GM on the other hand make assumptions concerning the timing and speed of technological growth which they claim are "consistent with historical evidence." Specifically, they assume that 1) modernization either in agriculture or in manufacturing is not initially feasible, 2) modernization occurs first in the agricultural sector, and 3) growth in industrial-sector productivity is faster than growth in agricultural-sector productivity. Compelling as these assumptions are to us, they are by no means universally held.[3] To give stronger credence to the so-called "Crafts-Harley view" that early industrialization was confined to just a few industries (Crafts and Harley 1992), we form a technological growth model that can *endogenously* mimic these historical trends.

Finally, rather than suddenly open up the North and South to trade, we allow for *gradual* increases in North-South commerce. The British economy (and other Western economies) presumably did not undergo a discontinuous switch from a closed to an open state, and thus we impose continuously declining transport costs to achieve such a transition.

These key differences allow us to conclude something that is somewhat under-explored in GM - growth in trade and technologies *together* created the dramatic divergence in living standards between the North and the South. More specifically, while the South's specialization in agriculture and low-end manufacturing allowed for plenty of technological advances in these areas, they did not help the South grow for two reasons. One is that it fostered fertility increases similar to the process outlined in GM. The other is that the South's terms of trade deteriorated over time. As the South grew in population it increasingly made up a larger share of the *world* population. It thus flooded the world markets with its primary products. The skill-intensive products provided by the North on the other hand became relatively more scarce, and thus fetched higher prices. The South had to provide more and more primary products to buy the same amount of high-end products; this served to raise fertility rates by even more. This mechanism, absent in

[2]We know that Heckscher-Ohlin oriented trade was important during the 19th century since commodity price convergence induced factor price convergence during this time (O'Rourke and Williamson 1994; O'Rourke, Taylor and Williamson 1996; O'Rourke and Williamson 1999, Chapter 4). And Mitchener and Yan (2010) suggest that unskilled-labor abundant China exported more unskilled-labor intensive goods and imported more skill-intensive goods from 1903 to 1928, consistent with such a trade model.

[3]See for example Temin (1997) who suggests that the traditional view of the British Industrial Revolution as a broad change that affected all industries is still in high regard among some economic historians.

GM's work, suggests that productivity growth (and the scale that generated this growth) could not salvage the South; it in fact contributed to its relative decline.

To further examine the interactions between trade and technologies, we run counter-factual simulations where either technological growth in both regions is not possible, or trade between the two regions is not possible. Divergence in either case is minuscule compared with the case where trade and technological progress interact with each other. Both factors worked in tandem to generate the massive wealth disparity we see in the world today.

2 Production with Given Technologies and Factors

We now sketch out a model that we will use to describe both a northern economy and a southern economy (superscripts denoting region are suppressed for the time being).

Total production for a region is given by:

$$Y = \left(\frac{\alpha}{2} y_1^{\frac{\sigma-1}{\sigma}} + (1-\alpha) y_2^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2} y_3^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $\alpha \in [0,1]$ and $\sigma \geq 0$. σ is the elasticity of substitution among intermediate goods y_1, y_2, and y_3. The production of these goods are given by:

$$y_1 = A_1 L_1 \tag{2}$$

$$y_1 = A_2 L_2^{\gamma} H_2^{1-\gamma} \tag{3}$$

$$y_3 = A_3 H_3 \tag{4}$$

where A_1, A_2 and A_3 are the technological levels of sectors 1, 2, and 3, respectively.[4] These technological levels in turn are represented by a series of *sector-specific* machines. Specifically,

$$A_1 = \int_0^{N_1} \left(\frac{x_1(j)}{L_1}\right)^{\alpha} dj \tag{5}$$

$$A_2 = \int_0^{N_2} \left(\frac{x_2(j)}{L_2^{\gamma} H_2^{1-\gamma}}\right)^{\alpha} dj \tag{6}$$

$$A_3 = \int_0^{N_3} \left(\frac{x_3(j)}{H_3}\right)^{\alpha} dj \tag{7}$$

[4] Thus sectors vary by *skill*-intensity. While our interest is mainly in the "extreme" sectors (1 and 3), we require an intermediate sector so that production of intermediate goods are determined both by relative prices and endowments, and not pre-determined solely by endowments of L and H. This will be important when we introduce trade to the model.

where $x_i(j)$ is machine of type j that can be employed only in sector i. Intermediate producers choose the amounts of these machines to employ, but the number of *types* of machines in each sector is exogenous to producers. Technological progress in sector i can then be represented by growth in the number of machine-types for the sector, denoted as N_i (we endogenize the growth of these in the next sections by introducing researchers).

Treating technological coefficients as exogenous for the time being, we can assume that markets for both the final good and intermediate goods are perfectly competitive. Thus, prices are equal to unit costs. Solving the cost minimization problems for productions, and normalizing the price of final output to one, yields the unit cost functions

$$1 = \left[\left(\frac{\alpha}{2}\right)^{\sigma} (p_1)^{1-\sigma} + (1-\alpha)^{\sigma} (p_2)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma} (p_3)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{8}$$

$$p_1 = \frac{w_l}{A_1} \tag{9}$$

$$p_2 = \left(\frac{1}{A_2}\right) w_l^{\gamma} w_h^{1-\gamma} (1-\gamma)^{\gamma-1} \gamma^{-\gamma} \tag{10}$$

$$p_3 = \frac{w_h}{A_3} \tag{11}$$

where p_i denotes the price for intermediate good y_i, w_l is the wage paid to L and w_h is the wage paid to H.

Full employment of total unskilled labor and total skilled labor implies the following factor-market clearing conditions:

$$L = \frac{y_1}{A_1} + \frac{w_l^{\gamma-1} w_h^{1-\gamma} (1-\gamma)^{\gamma-1} \gamma^{1-\gamma} y_2}{A_2} \tag{12}$$

$$H = \frac{w_l^{\gamma} w_h^{-\gamma} (1-\gamma)^{\gamma} \gamma^{-\gamma} y_2}{A_2} + \frac{y_3}{A_3} \tag{13}$$

Finally, the demands for intermediate goods from final producers can be derived from a standard C.E.S. objective function.[5] Specifically, intermediate goods market clearing requires

$$y_i = \left(\frac{\Upsilon_i^{\sigma} p_i^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma} (p_1)^{1-\sigma} + (1-\alpha)^{\sigma} (p_2)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma} (p_3)^{1-\sigma}} \right) Y \tag{14}$$

for $i = 1, 2, 3$, $\Upsilon_1 = \Upsilon_3 = \alpha/2$, and $\Upsilon_2 = 1 - \alpha$.

Provided that we have values for L, H, A_1, A_2 and A_3, along with parameter values, this yields thirteen equations [(1) - (4), (8) - (13), and three versions of (14)] with thirteen unknowns [Y, p_1, p_2, p_3, y_1, y_2, y_3, w_l and w_h, L_1, L_2, H_2, and H_3]. The solution for these variables

[5] Here demands will be negatively related to own price, will be a function of a price index, and will be proportional to total product.

constitutes the solution for the *static* model in the case of exogenously determined technological and demographic variables.

3 Endogenizing Technologies in Both Regions

In this section we describe how innovators in both the North and the South endogenously develop new technologies. In general, modeling purposive research and development effort is challenging when prices and factors change over time. This is because it is typically assumed that the gains from innovation will flow to the innovator throughout his lifetime, and this flow will often depend on the price of the product being produced and the factors required for production at each moment in time.[6] If prices and factors are constantly changing (as they will in any economy where trade barriers fall gradually or factors evolve endogenously), a calculation of the true discounted profits from an invention may be impossibly complicated.

To avoid such needless complication but still gain from the insights of endogenous growth theory, we assume that the gains from innovation last *one time period only.* More specifically, technological progress is sector-specific, and comes about though increases in the varieties of machines employed in each sector. New varieties of machines are developed by profit-maximizing inventors, who *monopolistically* produce and sell the machines to competitive producers of the intermediate goods y_1, y_2 or y_3. However, we assume the blueprints to these machines become public knowledge the time period after the machine is invented, at which point these machines become old and are *competitively* produced and sold.[7] Thus while we need to distinguish between old and new sector-specific machines, we avoid complicated dynamic programming problems inherent in multiple-period profit streams.[8]

Thus, we can re-define sector-specific technological levels given by (5) - (7) as a series of both old and new machines at time t (once again suppressing region superscripts) as:

$$A_{1,t} = \left(\int_0^{N_{1,t-1}} x_{1,old}(j)^\alpha dj + \int_{N_{1,t-1}}^{N_{1,t}} x_{1,new}(j)^\alpha dj \right) \left(\frac{1}{L_1} \right)^\alpha$$

$$A_{2,t} = \left(\int_0^{N_{2,t-1}} x_{2,old}(j)^\alpha dj + \int_{N_{2,t-1}}^{N_{2,t}} x_{2,new}(j)^\alpha dj \right) \left(\frac{1}{L_2^\gamma H_2^{1-\gamma}} \right)^\alpha$$

[6]For example, the seminal Romer (1990) model describes the discounted present value of a new invention as a positive function of $L - L_R$, where L is the total workforce and L_R are the number of researchers. Calculating this value function is fairly straight-forward if labor supplies of production workers and researchers are constant. If they are not, however, calculating the true benefits to the inventor may be difficult.

[7]Here one can assume either that patent protection for intellectual property lasts one time period, or that it takes one time period for potential competitors to reverse-engineer the blueprints for new machines.

[8]See Rahman (2009) for more discussion of this simplifying (but arguably more realistic) assumption.

$$A_{3,t} = \left(\int_0^{N_{3,t-1}} x_{3,old}(j)^\alpha dj + \int_{N_{3,t-1}}^{N_{3,t}} x_{3,new}(j)^\alpha dj \right) \left(\frac{1}{H_3} \right)^\alpha$$

where $x_{i,old}$ are machines invented before t, and $x_{i,new}$ are machines invented at t. Thus in each sector i there are N_{t-1} varieties of old machines that are competitively produced, and there are $N_t - N_{t-1}$ varieties of new machines that are monopolistically produced (again, suppressing country subscripts).

Next, we must describe producers of intermediate goods in each region. These three different groups of producers each separately solve the following maximization problems:

Sector 1 producers: $\max_{[L_1, x_1(j)]} p_1 y_1 - w_l L_1 - \int_0^{N_1} \chi_1(j) x_1(j) dj$

Sector 2 producers: $\max_{[L_2, H_2, x_2(j)]} p_2 y_2 - w_l L_2 - w_h H_2 - \int_0^{N_2} \chi_2(j) x_2(j) dj$

Sector 3 producers: $\max_{[H_3, x_3(j)]} p_3 y_3 - w_h H_3 - \int_0^{N_3} \chi_3(j) x_3(j) dj$

where $\chi_i(j)$ is the price of machine j employed in sector i. For each type of producer, solving the maximization problem with respect to machine j yields a solution for machine demand:

$$x_1(j) = \chi_1(j)^{\frac{1}{\alpha-1}} (\alpha p_1)^{\frac{1}{1-\alpha}} L_1 \tag{15}$$

$$x_2(j) = \chi_2(j)^{\frac{1}{\alpha-1}} (\alpha p_2)^{\frac{1}{1-\alpha}} L_2^\gamma H_2^{1-\gamma} \tag{16}$$

$$x_3(j) = \chi_3(j)^{\frac{1}{\alpha-1}} (\alpha p_3)^{\frac{1}{1-\alpha}} H_3 \tag{17}$$

New machine producers, having the sole right to produce the machine, set the price of their machines to maximize instantaneous profit. This price will be a constant markup over the marginal cost of producing a machine. Assuming that the cost of making a machine is unitary implies that $\chi_1(j) = \chi_2(j) = \chi_3(j) = \chi = 1/\alpha$ for new machines. Thus, substituting in this mark-up price, and realizing that instantaneous profits are $(1/\alpha) - 1$ multiplied by the number of new machines sold, instantaneous revenues by new machine producers are given by:

$$\pi_1 = \left(\frac{1-\alpha}{\alpha} \right) \alpha^{\frac{2}{1-\alpha}} (p_1)^{\frac{1}{1-\alpha}} L_1 \tag{18}$$

$$\pi_2 = \left(\frac{1-\alpha}{\alpha} \right) \alpha^{\frac{2}{1-\alpha}} (p_2)^{\frac{1}{1-\alpha}} L_2^\gamma H_2^{1-\gamma} \tag{19}$$

$$\pi_3 = \left(\frac{1-\alpha}{\alpha} \right) \alpha^{\frac{2}{1-\alpha}} (p_3)^{\frac{1}{1-\alpha}} H_3 \tag{20}$$

Old machines, on the other hand, are competitively produced; competition will drive the price of all these machines down to marginal cost, so that $\chi_1(j) = \chi_2(j) = \chi_3(j) = \chi = 1$ for all old machines. Sectoral productivities can then be expressed simply as a combination of old and new machines demanded by producers. Plugging in the appropriate machine prices into our machine demand expressions (15) - (17), and plugging these machine demands into our sectoral productivities, we can express these productivities as:

$$A_1 = \left(N_{1,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{1,t} - N_{1,t-1}\right)\right) (\alpha p_1)^{\frac{\alpha}{1-\alpha}} \tag{21}$$

$$A_2 = \left(N_{2,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{2,t} - N_{2,t-1}\right)\right) (\alpha p_2)^{\frac{\alpha}{1-\alpha}} \tag{22}$$

$$A_3 = \left(N_{3,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{3,t} - N_{3,t-1}\right)\right) (\alpha p_3)^{\frac{\alpha}{1-\alpha}} \tag{23}$$

Thus, if we have given to us the number of old and new machines that can be used in each sector (the evolution of these are described in section 5.1 , we can simultaneously solve equations (8) - (14) and (21) - (23) to solve for prices, wages, intermediate goods and technological levels for a hypothetical economy. Our next goal then is to also endogenize the levels of skilled and unskilled labor in this hypothetical economy.

4 Endogenizing Population and Labor-Types in Both Regions

We now introduce an over-lapping generations framework, where individuals in each region live for two time periods. In their youths individuals work as unskilled workers; this income is consumed by their parents. When they become adults, individuals decide whether or not to expend a fixed resource cost to become a skilled worker. Adults also decide how many of their own children to have, who earn unskilled income for the adults. Adults however are required to forgo some income for child-rearing.

Specifically, an adult i's objective is to maximize current-period income. If an adult chooses to remain an unskilled worker (L), she aims to maximize I_l with respect to her number of children, where

$$I_l = w_l + n_l w_l - w_l \lambda n_l^{\phi} \tag{24}$$

w_l is the unskilled labor wage, n_l is the number of children that the unskilled adult has, and $\lambda > 0$ and $\phi > 1$ are constant parameters that affect the opportunity costs to child-rearing..

If an adult chooses to spend resources to become a skilled worker, she instead maximizes I_h with respect to her number of children, where

$$I_h = w_h + n_h w_l - w_h \lambda n_h^{\phi} - \tau_i \tag{25}$$

w_h is the skilled labor wage, n_h is the number of children that the skilled adult has, and τ_i is the resources she must spend to become skilled.

The first order conditions for each of these groups give us the optimal fertility for each group, n_l^* and n_h^*:

$$n_l^* = (\phi\lambda)^{\frac{1}{1-\phi}} \tag{26}$$

$$n_h^* = \left(\frac{w_h}{w_l}\phi\lambda\right)^{\frac{1}{1-\phi}} \tag{27}$$

Note that with $w_h > w_l$, the optimal fertility for a skilled worker is always smaller than that for an unskilled worker (this is simply because the opportunity costs of child-rearing are larger for skilled workers). Also note that the fertility for unskilled workers is constant, while the fertility for skilled workers falls with increases in the skill premium.

Finally, assume that τ varies across adults. The resource costs necessary to acquire an education can vary across individuals for many reasons, including differing incomes, access to schooling, or innate abilities. Say τ_i is uniformly distributed across $[0, b]$, where $b > 0$. An individual i who draws a particular τ_i will choose to become a skilled worker only if her optimized income as a skilled worker will be larger than her optimized income as an unskilled worker. Let us call τ^* the *threshold* cost to education; this is the education cost where the adult is indifferent between becoming a skilled worker or remaining an unskilled worker. Solving for this, we get

$$\tau^* = w_h + n_h^* w_l - w_h \lambda {n_h^*}^{\phi} - w_l - n_l^* w_l + w_l \lambda {n_l^*}^{\phi} \tag{28}$$

Only individuals whose τ_i fall below this level will opt to become skilled.

Figure 4 illustrate optimal fertility rates for two individuals - one with a relatively high τ and one with a relatively low τ. The straight lines illustrate how earnings increase as adults have more children; the slope of these lines is simply the unskilled wage w_l. The earnings line for a skilled worker is shifted up to show that she earns a premium. Cost curves get steeper with more children since $\phi > 1$. For skilled individuals, the cost curve is both higher (to illustrate the resource costs τ necessary to become skilled) and steeper (to illustrate the higher opportunity cost to have children). Notice then that the only difference between the high-τ individual and the low-τ individual is that the latter has a lower cost curve. The optimal fertility rates however are the same for both types of adults. Given these differences in the fixed costs of education, we can see that the high-τ individual will opt to remain an unskilled worker (and so have a fertility rate of n_l^*), while the low-τ individual will choose to become skilled (and have a fertility rate of n_h^*).

With this we can describe aggregate supplies of skilled and unskilled labor (demands for these labor types are described by full employment conditions (12) and (13)), fertility and education. Given a total adult population equal to *pop*, we can describe these variables as:

$$H = \left(\frac{\tau^*}{b}\right) pop \tag{29}$$

$$L = \left(1 - \frac{\tau^*}{b}\right) pop + npop \tag{30}$$

$$n = \left(1 - \frac{\tau^*}{b}\right) n_l^* + \left(\frac{\tau^*}{b}\right) n_h^* \tag{31}$$

$$e = \frac{\tau^*}{b} \tag{32}$$

where H is the number of skilled workers (comprised strictly of old workers), L is the number of unskilled workers (comprised of both old and young workers), n is aggregate fertility, e is the fraction of the workforce that gets an education, and n_l^*, n_h^*, and τ^* are the optimized fertility rates and threshold education cost given respectively by (26), (27) and (28).

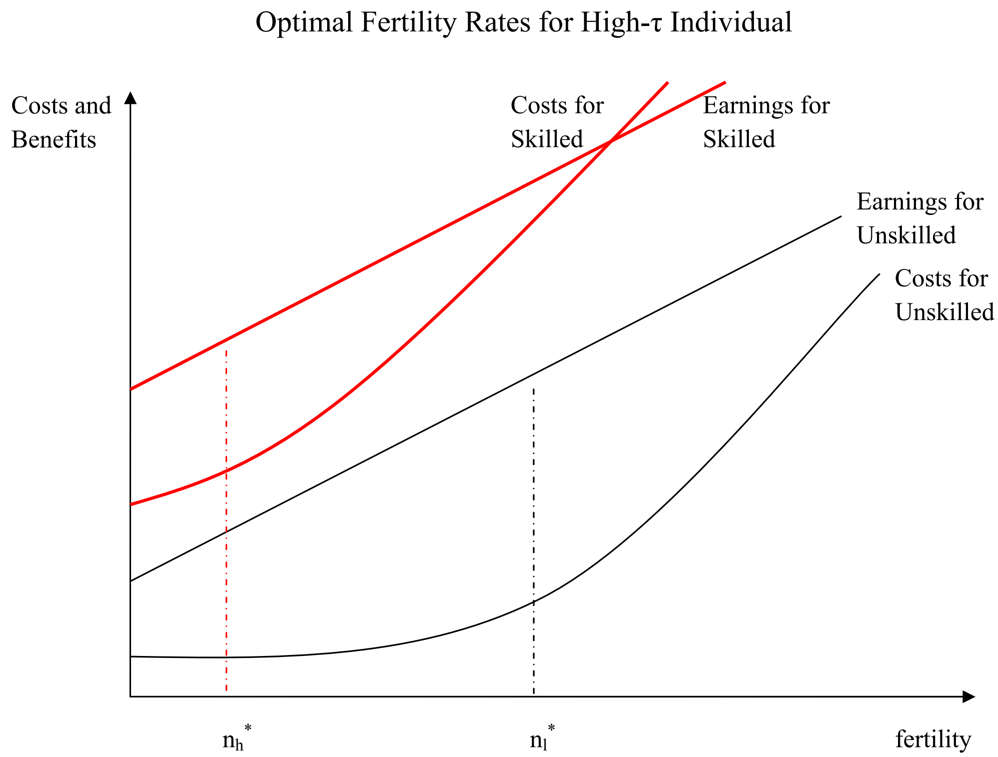
This completes the description of the static one-country model. The next section uses this model to describe *two* economies that endogenously develop technologies and trade with each other to motivate a story of world economic history.

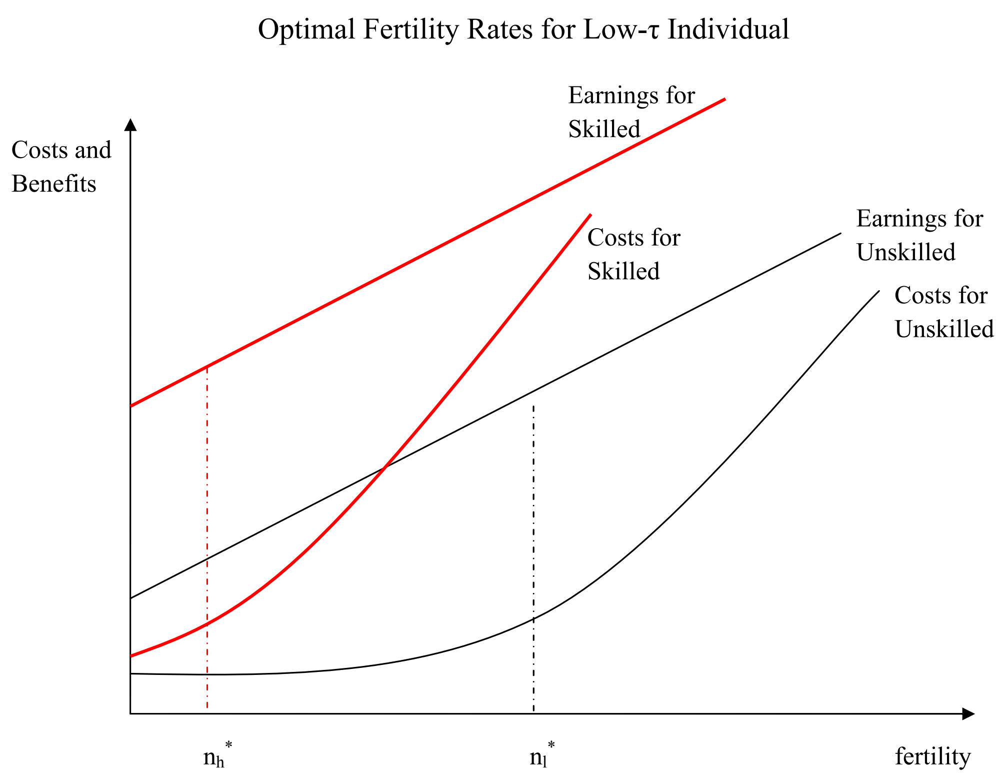
5 The Roles of Trade and Technological Growth in the Great Divergence

In this section we show how the interactions between the growth of trade and biased technologies contributed to the great divergence of the late 19th Century. To do this we perform a thought experiment by simulating two economies. The above model describes a hypothetical country - now we will use it to describe both a “northern” economy and a “southern” economy, where the southern economy is relatively more unskilled labor-endowed.

The simulations demonstrate a number of things. Early industrialization in both regions was unskilled labor intensive (O’Rourke et al 2008). Trade between the two regions generates some income convergence early on - specialization induces the North to devote R&D resources to the skill-intensive sector and the South to devote resources to the unskilled-intensive sector. Because the skilled sector is so much smaller than the unskilled sector, the South is able to grow relatively faster at first. But the dynamic effects of these growth paths (through fertility and education changes) ultimately reverses income convergence. The reinforcing interactions between

Figure 4: Optimal Fertility Rates for High and Low τ Individuals (for given w_l and w_h)





technological growth and intercontinental commerce help produce dramatic divergence between the incomes of northern and southern economies.

To simulate this tale however, we will first need to endogenize the time paths of technologies and trade volumes.

5.1 A Dynamic Model - The Evolution of Technology and Trade

How do technologies grow in each region? Recall that equations (18) - (20) describe one-period revenues for innovation. There must also be some resource costs to research. For this, we can assume that these costs are rising in N ("applied" knowledge, specific to each sector and to each country), and falling in some measure of "general" knowledge, given by B (basic knowledge, common across all sectors and countries). Thus, a no-arbitrage (free entry) condition for potential researchers in each region can be described as:

$$\pi_i \leq c\left(\frac{N_i}{B}\right) \tag{33}$$

Specifically, we can assume the following functional form for these research costs:

$$c\left(\frac{N_i}{B}\right) = \left(\frac{N_{i,t+1}}{B_t}\right)^{\nu} \tag{34}$$

for $i = 1, 3$ (for convenience we assume no research occurs in sector 2. This way technological growth is unambiguously factor-biased), and $\nu > 0$. Given some level of basic knowledge (which we can assume grows at some exogenous rate) and number of existing machines, we can determine the resource costs of research. When basic knowledge is low relative to the number of available machine-types used in sector i, the costs of inventing a new machine in sector i is high (see O'Rourke et al. 2008 for a fuller discussion). Thus from (33) and (18) - (20) we see that innovation in sector i becomes more attractive when basic knowledge is large, when the number of machine-types in sector i is low, when then price of good i is high, and when the employment in sector i is high.[9]

Note that if $\pi_i > c(N_i/B)$, there are potential profits from research in sector i. However, this will induce research activity, increasing the number of new machines, and hence costs of research, up. We assume in fact that N_i adjusts upward such that costs of research just offset the revenues of new machine production. Thus increases in B are matched by increases in levels of N_i such that the no-arbitrage condition holds with equality whenever technological growth in the sector occurs.

[9] For ease of analysis we assume that the numbers of blueprints in each region are independent of each other. We relax this assumption in section 5.4.

We must also specify how trade technologies evolve. Here we use an amended version of (1), where production for each region is given by

$$Y^n = \left(\frac{\alpha}{2}\left(y_1^n + aZ_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(y_2^n\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(y_3^n - Z_3\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{35}$$

$$Y^s = \left(\frac{\alpha}{2}\left(y_1^s - Z_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(y_2^s\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(y_3^s + aZ_3\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{36}$$

Z_1 is the amount of good 1 that is exported by the South, Z_3 is the amount of good 3 that is exported by the North, and $0 < a < 1$ is an iceberg factor for traded goods (i.e. the proportion of exports not lost in transit). Thus the North imports only fraction a of southern exports, and the South imports only fraction a of northern exports.[10] Intermediate goods production is still described by (2) - (4). To capture improvements in transport technologies over the course of the 18th and 19th centuries, we simply have a grow exogenously each time period, such that it reaches the limiting value of 1 by the end of the simulation.

Note that we assume that there is no trade in y_2 - because this is produced using both L and H, differences in p_2 are very small between the North and the South, and thus the assumption is not very restrictive or important.[11] [12]

5.2 Evolution of the World Economy

General equilibrium is a thirty-six equation system that, given changes in the number of machine blueprints and the iceberg costs, solves for prices, wages, fertility, education, labor-types, intermediate goods, employment, trade, and sectoral productivity levels for both the North and the South. We impose only one parameter difference between the two regions - $b^n < b^s$ (this means that there is a bigger range of resource costs for education in the South, so that the South begins with relatively more unskilled labor than skilled labor). All other parameters are the same in both regions. Fertility is normalized to one in the beginning for each country, so that population is stable. The equilibrium is described in more detail in the appendix.

Because the model contains so many moving parts, we can only solve for general equilibrium numerically. Specifically, we assume that both basic technology (B in eq 34) and trade technology

[10] The case where the North specializes in and exports the *unskilled*-intensive good and the South specializes in and exports the *skilled*-intensive good is ruled out due to the North's relative abundance of skilled labor. The North would have to have very high levels of unskilled-biased technology compared to the South to reverse its comparative advantage in skill-intensive production.

[11] Indeed, trade in all three goods would produce an analytical problem. It is well known among trade economists that when there are more traded goods than factors of production, country-specific production levels, and hence trade volumes, are indeterminate. See Melvin (1968) for a thorough discussion.

[12] One can conceive of y_2 as the technologically-stagnant and non-tradeable "service" sector. Thus each labor-type can work either in manufacturing or in services.

(a in eqs 35 and 36) start low enough so that neither technological progress nor trade are possible. We allow however for exogenous growth in basic knowledge and trade technologies, and solve for the endogenous variables each period. Let us first summarize the evolution of these two economies with a few propositions, starting with the nature of early industrialization in the world.

Proposition 1 *If $N_1 = N_3$, $L > H$, and $\sigma > 1$, initial technological growth will be unskilled-labor biased.*

From (18)-(20) we can see that revenues from innovation rise both in the price of the intermediate good (the "price effect") and in the scale of sectoral employment (the "market-size effect). If intermediate goods are grossly substitutable, market-size effects will outweigh price effects (see Acemoglu 2002 for more discussion of this).

Thus as basic knowledge exogenously grows, sector 1 will be the first to modernize. The logical implication of this is that early industrialization around the world (provided there are intellectual property rights in these countries) will be unskilled labor intensive (O'Rourke et al. 2008).

Proposition 2 *If $\left(\frac{p_3^n}{p_3^s}\right) \cdot \left(\frac{p_1^s}{p_1^n}\right) > a^2$, $Z_1 = Z_3 = 0$.*

If transport costs are large (that is, if a is small) relative to cross-country price differences, no trade occurs. As mentioned above, we will assume that early on transport technologies are not advanced enough to permit trade. That is, with a small value of a, each country can produce more under autarky than by trading. Once a reaches this threshold level, trade becomes possible, and further increases in a allows Z_1 and Z_3 to rise as well.

Proposition 3 *For certain ranges of factors and technologies, the trade equilibrium implies that $y_3^s = 0$. For other ranges of technologies and factors, the trade equilibrium implies that $y_1^n = 0$.*

As trade technologies improve, economies specialize more and more. And divergent technological growth paths can help reinforce this specialization. There is indeed a point where the North no longer needs to produce any y_1 (they just import it from the South), and the South no longer needs to produce any y_3 (they just import it from the North). This case we will call the "specialized trade equilibrium" (described in more detail in the Appendix).

Both trade and technological changes will change factor payments. The final proposition states how these changes can affect the factors of production themselves.

Proposition 4 *If $\phi > 1$, any increase in w_l (keeping w_h constant) will induce a decrease in e and an increase in n; furthermore, so long as ϕ is "big enough," any increase in w_h (keeping w_l constant) will induce an increase in e and a decrease in n.*

Proof.

Substituting our expressions for n_l^* and n_h^*, given by (26) and (27), into our expression for τ^*, given by (28), and rearranging terms a bit, we get the following expression:

$$\tau^* = (w_h - w_l) \; - \; w_l \lambda^{\frac{1}{1-\phi}} \left(\phi^{\frac{1}{1-\phi}} - \phi^{\frac{\phi}{1-\phi}} \right) \; + \; wl^{\frac{\phi}{\phi-1}} wh^{\frac{1}{1-\phi}} \lambda^{\frac{1}{1-\phi}} \left(\phi^{\frac{1}{1-\phi}} - \phi^{\frac{\phi}{1-\phi}} \right)$$

First we must have the condition $\frac{\partial \tau^*}{\partial w_l} < 0$ hold. Solving for this and rearranging yields

$$\left(\frac{w_l}{w_h} \right)^{\frac{1}{\phi-1}} < 1 + \frac{1}{\lambda^{\frac{1}{1-\phi}} \left(\phi^{\frac{1}{1-\phi}} - \phi^{\frac{\phi}{1-\phi}} \right)}$$

Since the inverse of the skill-premium is always less than one, this expression always holds for any $\phi > 1$. Next we show what condition must hold in order to have the expression $\frac{\partial \tau^*}{\partial w_h} > 0$ be true. Solving and rearranging gives us

$$\lambda^{\frac{1}{\phi}} \phi > \frac{w_l}{w_h}$$

Thus for a given value of λ, ϕ needs to be large enough for this condition to hold. Finally, our expression for total fertility, (31), can be slightly rearranged as

$$n = n_l^* + (n_h^* - n_l^*) \left(\frac{\tau^*}{b} \right)$$

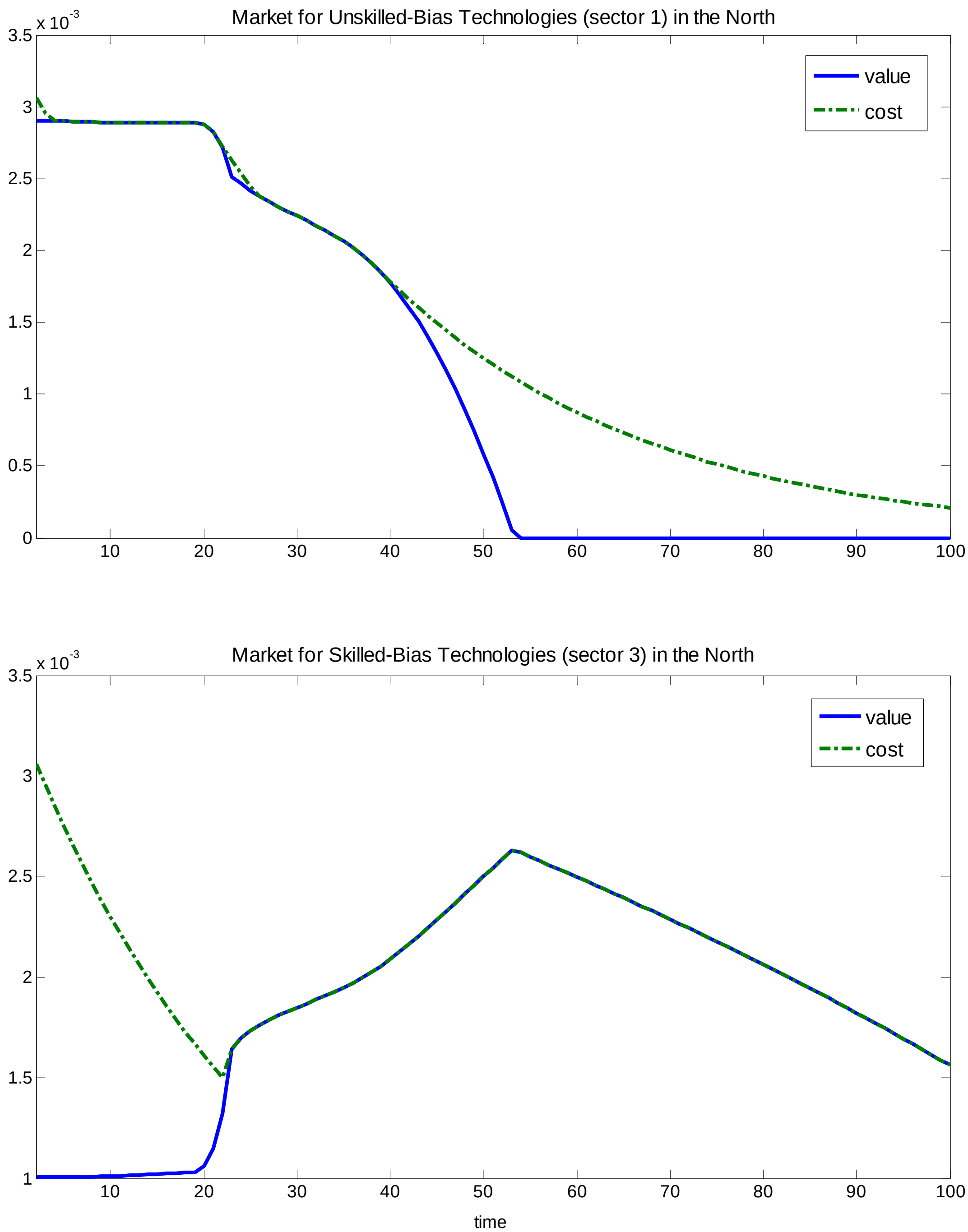
From (26) and (27) we know that the second term is always negative, and that n_l^* is constant. So any increase in education from wage changes will lower aggregate fertility, and any decrease in education from wage changes will increase aggregate fertility.

5.3 Simulations[13]

Here we simulate the model described above to analyze the potential sources of North-South divergence in history. Basic knowledge B and trade technology a are set such that neither technological growth nor trade is possible at first; each however exogenously rises over time. We run the simulation for 100 time periods to roughly capture major economic trends from around 1700 to the turn of the twentieth century.

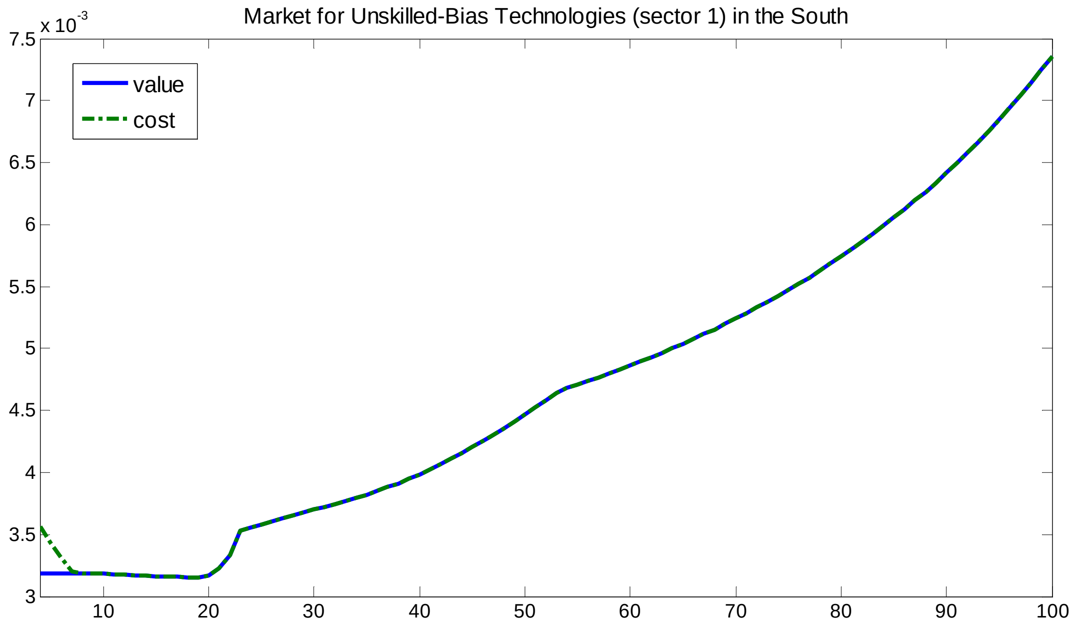
[13]The parameter values used in the simulations are as follows: $\sigma = 3$, $\alpha = 0.5$, $\gamma = 0.5$, $\lambda = 0.5$, $\phi = 10$, $\nu = 2$. These values ensure that Propositions 1 and 4 hold - beyond that, our qualitative findings are not sensitive to specific parameter values. We also set $b^n = 2$, $b^s = 6$, and $pop = 2$; this gives us initial factor endowments of $L_n = 3.14$, $L_s = 3.48$, $H_n = 0.86$, $H_s = 0.52$. Initial machine blueprints for both countries are set to be $N_1 = 10$, $N_2 = 15$, $N_3 = 10$; initial trade technology is set to be $a = 0.85$, and grows linearly such that $a = 1$ 100 periods later; initial B is set high enough so that growth in at least one sector is possible early in the simulation; B grows 2 percent each time period.

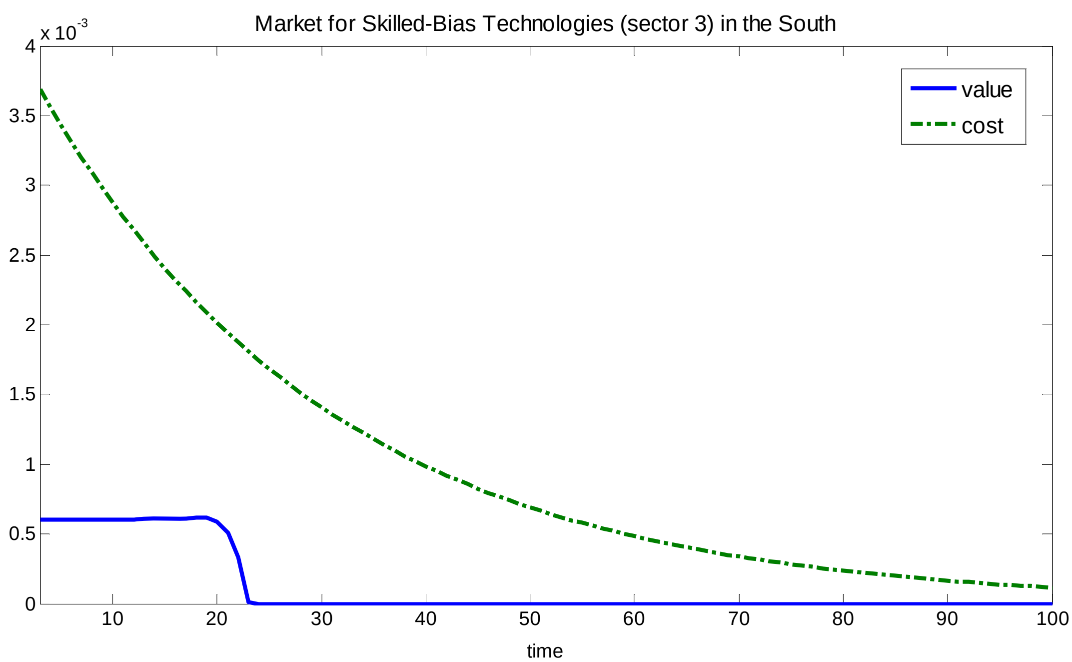
Figure 5: The Market for Technologies in the North



Note that because $L_n > H_n$ (combined with the fact that $\sigma > 1$), the revenues of new research in the unskilled sector is initially larger than those for the skilled sector. Consequently, unskilled knowledge is the first to grow (at $t = 3$). Trade (which becomes possible at $t = 20$) spurs innovation in the skilled sector since it raises both the price and employment level in sector 3, pulling research effort away from sector 1.

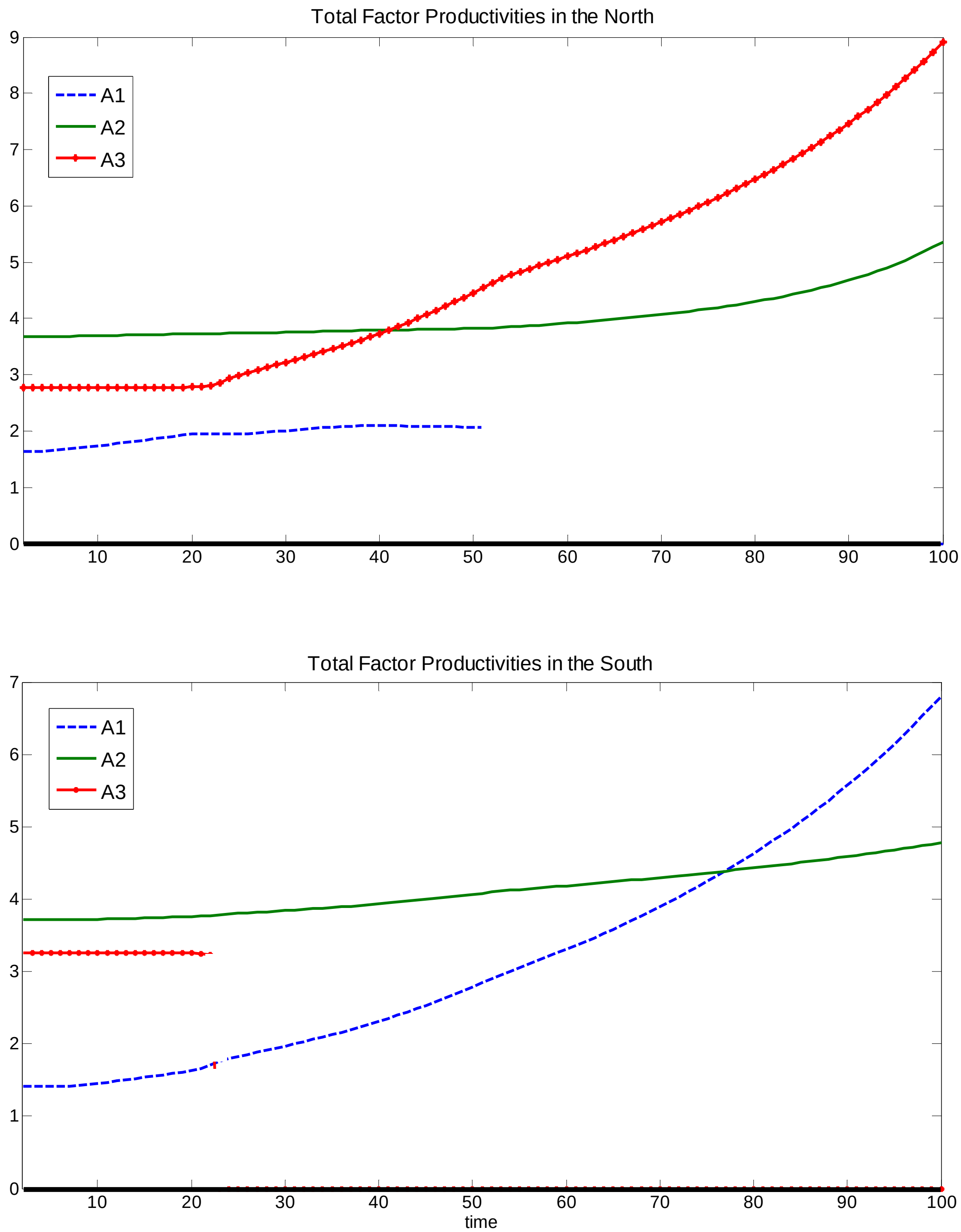
Figure 6: The Market for Technologies in the South





Just like the North, $L_s > H_s$, so unskilled technologies grow early in the simulation for the South as well. But unlike the North, trade reinforces this unskilled intensive growth. Trade also destroys the South's potential to develop skill-intensive technologies, since it helps eliminate sector 3 altogether! Consequently the South never obtains "balanced" technological growth.

Figure 7: Factor Productivities



Figures 5 through 7 summarize the evolution of technologies in both regions. In the beginning the costs of research are prohibitively high everywhere, so technologies are stagnant. But growth in basic knowledge allows us to see the implications of Proposition 1 - because there is a greater abundance of unskilled labor relative to skilled labor in both the North and the South, the costs of research first catch up to revenues in sector 1 in both regions.

This growth in unskilled labor intensive technologies lowers the relative returns to unskilled labor in both regions, inciting fertility increases and educational decreases (Proposition 4). We can see this manifest itself in the North by the increases in the revenues generated by innovation - as population rises in the North, the market-size effects caused by fertility increases raises the value of such innovation. Still, because skilled labor remains in relative scarce supply, the cost of innovation exceeds the benefits in sector 3 for the beginning of the simulation.

But evidently, looking at these figures, some event happens at $t = 20$ that in the North seems to pull resources away from sector 1 (so that potential revenues to research in the sector fall) and push resources into sector 3 (so that potential revenues in the sector rise). The opposite seems to happen in the South - resources are pulled from sector 3 to sector 1. Soon after this the North begins to develop new machine blueprints for skilled labor intensive sector 3. The South on the other hand increases its development of unskilled-intensive production, while abandoning its production of the sector 3 good altogether. What happened?

The answer is that at $t = 20$ the trade technology parameter a becomes large enough so that commerce between the two regions becomes possible (Proposition 2). At this point the South starts exporting some of its production of y_1 and the North starts exporting some of its production of y_3. Once such trade occurs, both price and market-size effects rise in sector 3, and innovation in the sector in the North begins.

The opposite happens in the South. Producing very little of good 3 even in autarky, the South finds itself importing all of the good from the North once the North raises its productivity in the sector (Proposition 3). Of course, this ultimately means that it will not be able to produce any skill-intensive innovations even with high B values (note that in the absence of trade such technological growth *would* have been possible at around $t \approx 50$).

Figures 8 and 9 chart the evolution of fertility, education, trade, and incomes per person in both regions. Early industrialization without any trade is characterized by rising fertility and falling education. Initial trade generates the demographic divergence that we would expect - we see continued increases in fertility and declines in education in the South, and a demographic transition of falling fertility and rising education in the North. This however produces some slight *convergence* in incomes per capita between the two regions. The reason, as mentioned earlier, is that sector 3 is smaller than sector 1 in both economies. Sector 3 suddenly becomes the modernizing sector in the North, while sector 1 continues to grow technologically in the South. Through scale effects, the South benefits more from this specialization; the growing sector in the

Figure 8: Rates of Fertility and Education

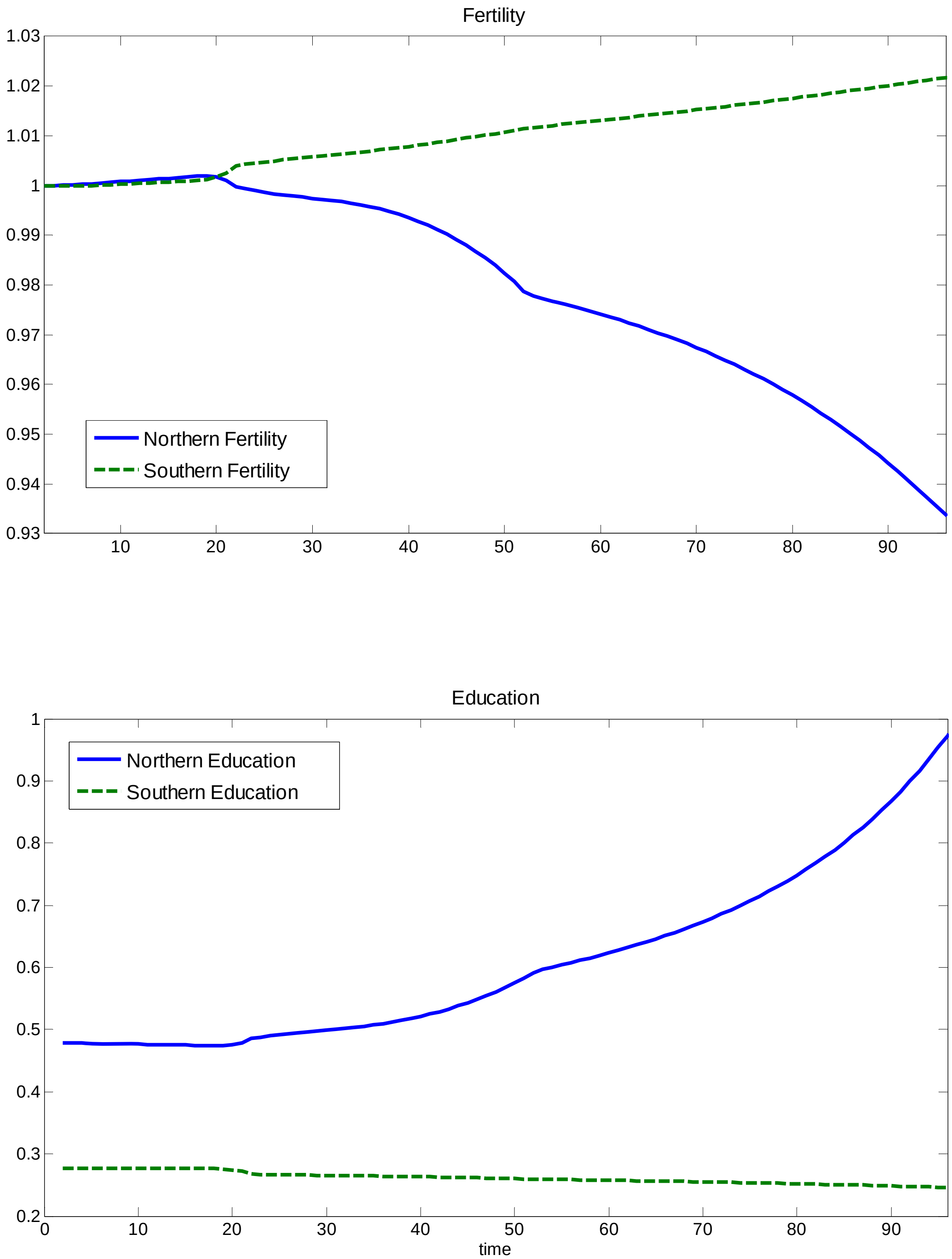
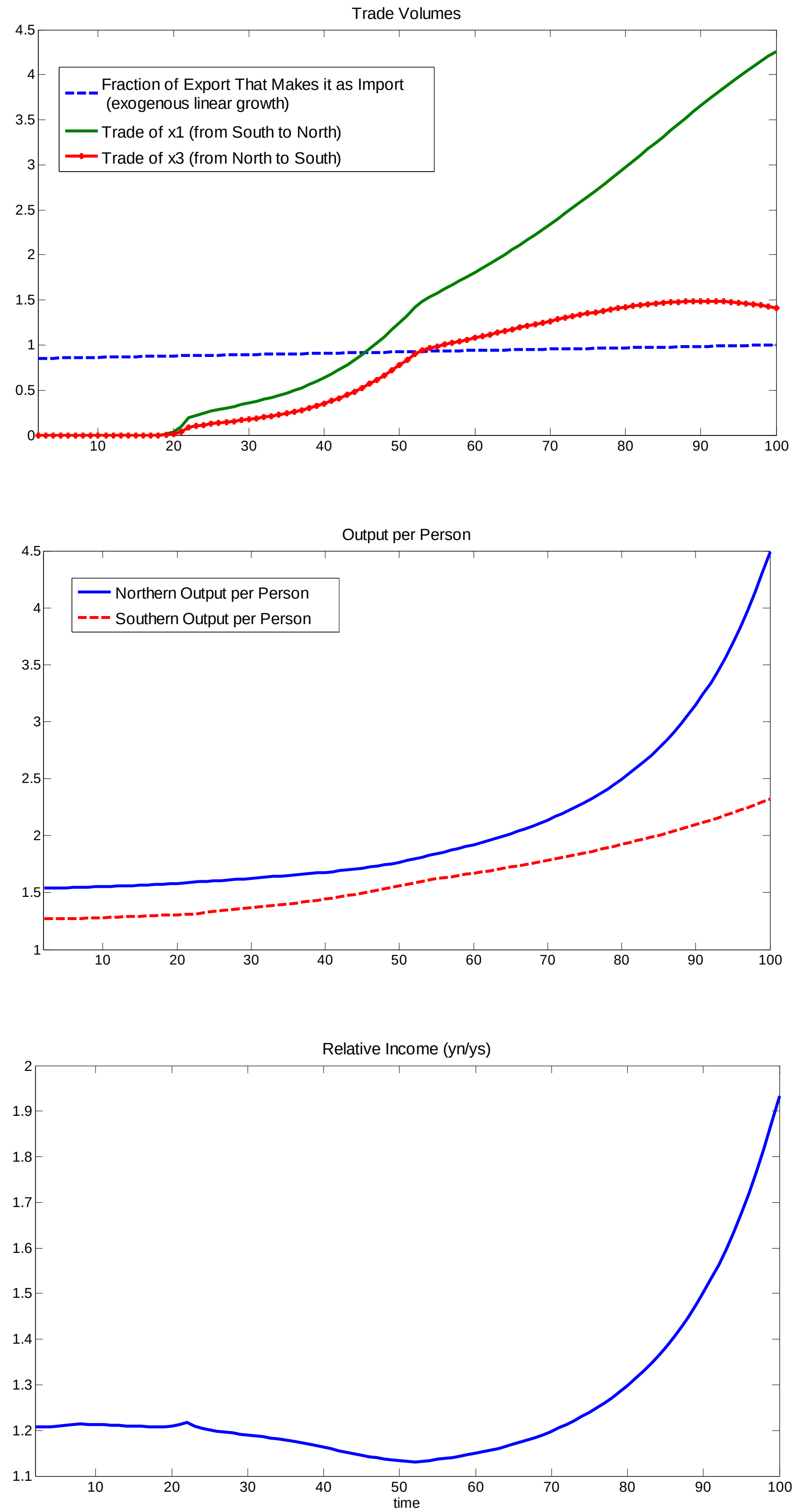


Figure 9: Trade and Output



North on the other hand is fairly small and cannot generate as much aggregate growth.

This trend towards convergence ultimately reverses with continued growth in trade and technologies. Specialization becomes so pronounced that the South quickly abandons production of good 3 altogether. This seals the South's fate forever as an unskilled-intensive producer. No matter how large basic knowledge grows, it can never generate new skill-intensive technologies, for the skill-intensive sector no longer exists for them. And the North eventually abandons its production of the unskilled good, reinforcing even further the South's energies in the production of this good.

Notice in figure 9 that once specialized trade occurs, the South's terms of trade begins to rapidly deteriorate - the South has to give up more and more y_1 for the same amount of y_3. Why is this? Because of its rapid population growth, the South increasingly makes up a larger share of the world population. It thus begins to flood the market with its primary products. The skill-intensive product on the other hand becomes relatively more scarce, and thus fetches a higher and higher price. Of course, all the y_1 that the South produces in order to acquire increasingly precious quantities of y_3 raises fertility even more, further eating away at per capita incomes.

Ultimately the increasing ability to trade mixed with biased technological growths in each country foster the divergence. There is rapid fertility declines in the North due to its rising production of good 3; the South on the other hand retains its high fertility due to the low skill premia generated by its specialization of good 1. The demographic transition of the North, and the lack thereof in the South, ultimately creates a great divergence in output per person between the two regions.[14]

5.4 Some Counterfactuals

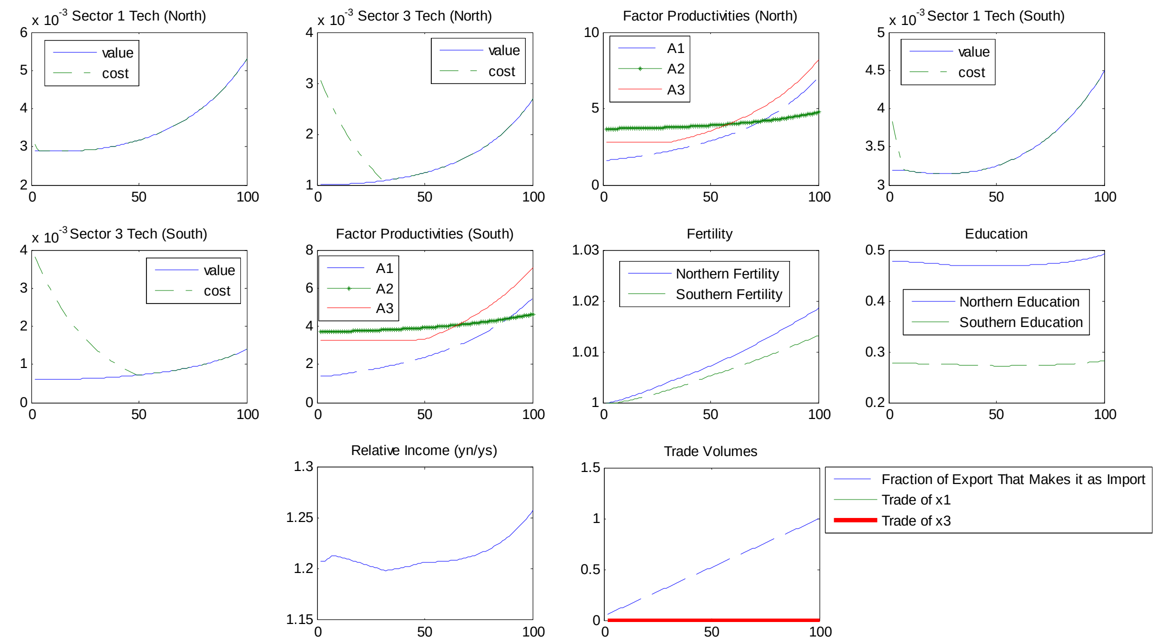
The above simulation stresses that both trade and technological forces interacting together created the divergence in living standards around the world. Here we perform some counterfactual experiments to further support this idea. We ask, what if we have an endogenous biased technological growth model, but do not allow for any trade between the North and the South? On the other hand, what if transport costs fall (just like our main simulation) so that trade eventually becomes possible, but technological progress can never occur?

Figure 10 displays the simulation results of these exercises. We can see that each factor by itself creates very modest divergence in living standards (measured by changes in y_n/y_s). Trade alone can not generate such divergence; the South's ability to develop unskilled-intensive technologies allows its *absolute* income level to grow, but through even more rapid population

[14]In fact, the demographic transition is so dramatic in the North that the shrinking population induces a drop in the scale of the market for inventors, slowing down Northern technological growth. In reality, immigration and other factors kept populations in northern countries from dropping so dramatically.

Figure 10: Simulations without Technology-Trade Interaction

No Trade



No Technological Growth

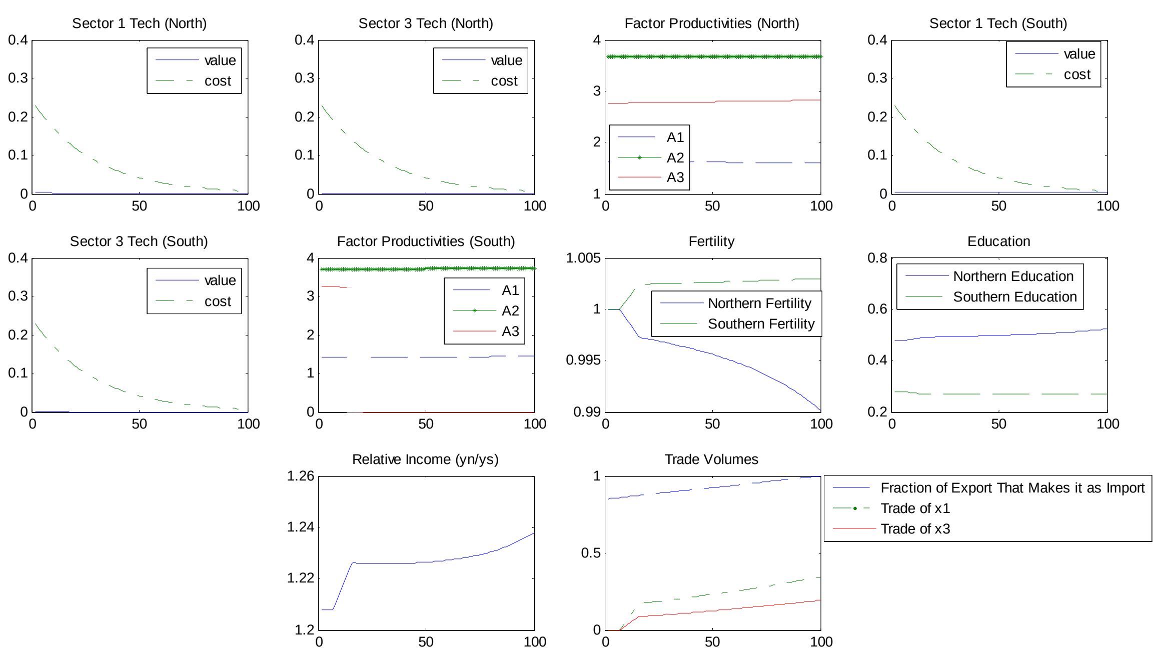
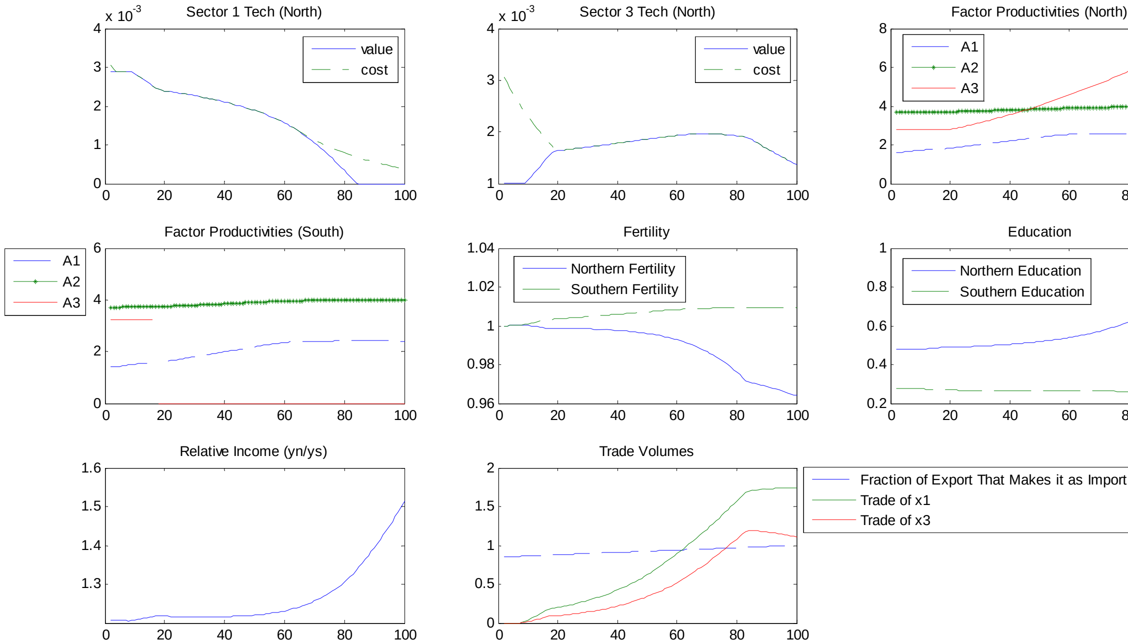
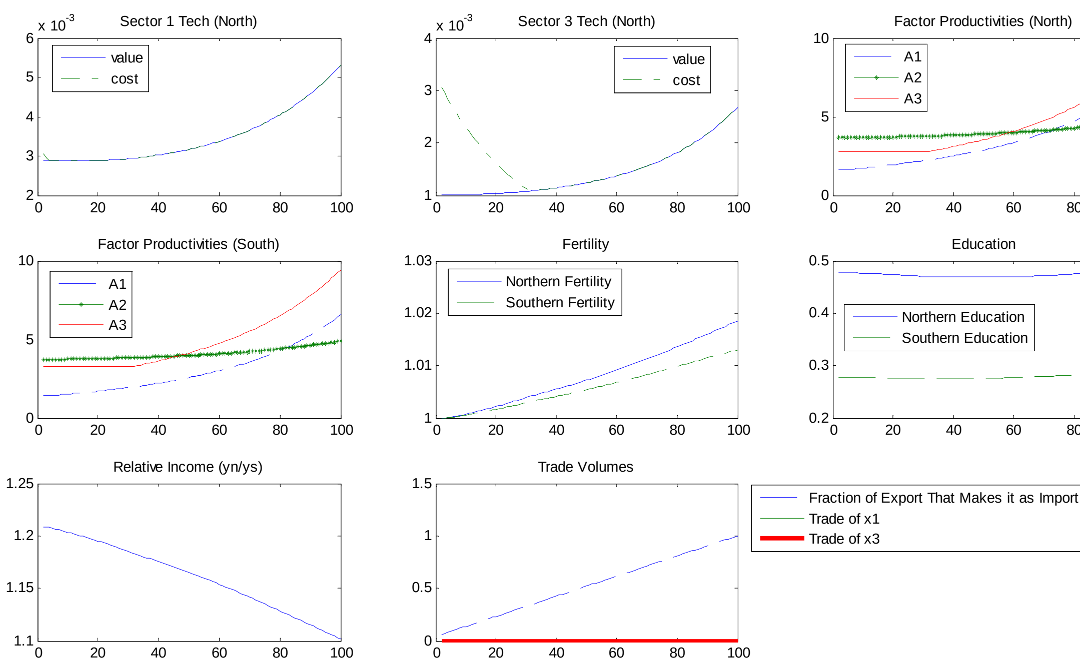


Figure 11: Simulations with Perfect Northern Technological Diffusion to the South

Technological Growth in North with Perfect Diffusion to the South; Regular Trade



Technological Growth in North with Perfect Diffusion to the South; No Trade



growth keeps its income *per capita* level anchored. Thus unlike the GM approach that has the South specialize in the *inherently* technologically-lagging sector, we generate a divergence even as the South endogenously experiences rapid technological progress.

Another hypothetical - what if the South could not develop technologies themselves, but instead relied on the diffusion of technologies from the North? Some could argue that this is a more realistic scenario, since the new technologies of the Industrial Revolution could be exported mechanically with relative ease to most of the world. After all, while developing new knowledge was an arduous task, copying this knowledge was much easier. This was particularly true of the technologies of early industrialization; since they were not very sophisticated, they were quickly transmitted to, and easily adopted by, much of the world (Mokyr 1999; Clark 2007, Chapter 15).

Figure 11 displays the results of simulations where the South instantly inherits the machine blueprints of the North (where $N^n_{1,t} = N^s_{1,t}$, $N^n_{2,t} = N^s_{2,t}$, and $N^n_{3,t} = N^s_{3,t}\ \forall\ t$). Again, we can see that the *interaction* between trade and technological developments creates the divergence (top figure); technological diffusion with no trade allowed actually generates convergence (bottom figure). Divergence in this case happens for a slightly different reason - the North develops skill-intensive technologies that the South can not use (it stops producing the skill-intensive good early in the simulation). Technological diffusion in this case is "inappropriate" - the receiving country does not have the appropriate factors needed to exploit such knowledge (see Basu and Weil 1998; Acemoglu and Zilibotti 2001). Notice however that without trade, the North never switches to skill-intensive production; consequently technological diffusion is always of the "appropriate" variety for the South (that is, unskilled intensive), and so income convergence occurs. Once again, we see trade is important in the story of divergence, but it is its interaction with biased technological developments that truly ripped open the chasm in living standards around the world.

References

Acemoglu, Daron. 2002. "Directed Technical Change," *Review of Economic Studies* 69: 781-809.

Acemoglu, Daron. forthcoming. "When Does Labor Scarcity Encourage Innovation?" *Journal of Political Economy*.

Acemoglu, Daron, Simon Johnson, and James A. Robinson. 2001. "The Colonial Origins of Comparative Development: An Empirical Investigation." *American Economic Review*, 91(5): 1369-1401.

Acemoglu, Daron, Simon Johnson, and James A. Robinson. 2005. "The Rise of Europe: Atlantic Trade, Institutional Change, and Economic Growth." *American Economic Review*, 95(3): 546-79.

Acemoglu, Daron and Fabrizio Zilibotti. 2001. "Productivity Differences," *Quarterly Journal of Economics* 116: 563-606.

Aghion, Philippe and Peter Howitt. 1992. "A Model of Growth through Creative Destruction," *Econo-*

metrica, 60: (323-351).

Basu, Susanto and David N. Weil. 1998. "Appropriate Technology and Growth," *Quarterly Journal of Economics* 113: 1025-1054.

Bekar, Clive and Richard Lipsey. 1997. "A Structuralist View of Technical Change and Economic Growth." Reprinted in *Microeconomics, Growth and Political Economy: The Selected Essays of Richard Lipsey Vol. One.* Edward Elgar: Cheltenham, UK.

Clark, Gregory. 2007. "A Review of Avner Grief's Institutions and the Path to the Modern Economy: Lessons from Medieval Trade." *Journal of Economic Literature*, XLV: 727-743.

Clark, Gregory. 2007. *A Farewell to Alms: A Brief Economic History of the World.* Princeton and Oxford: Princeton University Press.

Clark, Gregory, and Robert C. Feenstra. 2001. "Technology in the Great Divergence," in NBER book *Globalization in Historical Perspective.*

Clark, Gregory and Susan Wolcott. 2003. "One Polity, Many Countries: Economic Growth in India, 1873-2000." in Dani Rodrik (ed.), *Frontiers of Economic Growth.* Princeton: Princeton University Press

Crafts, N.F.R., and C.K. Harley. 1992. "Output Growth and the Industrial Revolution: A Restatement of the Crafts-Harley View," *Economic History Review* 45(4): 703-30.

Findlay, Ronald. 1996. "Modeling Global Interdependence: Centers, Peripheries, and Frontiers," *American Economic Review* 86(2): 47-51.

Galor, Oded. 2005. "From Stagnation to Growth: Unified Growth Theory," in *Handbook of Economic Growth*, Vol. 1, part 1, edited by P. Aghion and S.N. Durlauf, pp. 171-293. Amsterdam: North Holland.

Galor, Oded, and Andrew Mountford. 2002. "Why are one Third of People Indian and Chinese? Trade, Industrialization and Demographic Transition," manuscript.

Galor, Oded and Andrew Mountford. 2006. "Trade and the Great Divergence: The Family Connection," *American Economic Review Papers and Proceedings*, 96: 299-303.

Galor, Oded and Andrew Mountford. 2008. "Trade and the Great Divergence: Theory and Evidence,"*Review of Economic Studies*, 75: 1143-1179.

Grossman, Gene M. and Elhanan Helpmann. 1991. *Innovation and Growth in the Global Economy.* Cambridge, MIT Press.

Howitt, Peter. 1999. "Steady Endogenous Growth with Population and RD Inputs Growing," *Journal of Political Economy*, 107: 715-730.

Jones, Charles I. 1995. "R&D-Based Models of Economic Growth," *Journal of Political Economics*, 103: 759-784.

Melvin, James R. 1968. "Production and Trade with Two Factors and Three Goods," *American Economic Review* 58: 1249-1268.

Mitchener, Kris J., and Se Yan. 2010. "Globalization, Trade and Wages: What Does History Tell Us about China?" NBER working paper no. 15679.

Mokyr, Joel. 1999. *The British Industrial Revolution: An Economic Perspective*, 2nd edition. Westview Press: Boulder, CO.

Mokyr, Joel. 2002. *The Gifts of Athena*. Princeton, N.J.: Princeton University Press.

North, Douglass C., and Robert Paul Thomas. 1973. *The Rise of the Western World: A New Economic History*. Cambridge: Cambridge University Press.

O'Rourke, Kevin H., Ahmed S. Rahman and Alan M. Taylor. 2008. "Luddites and the Demographic Transition," NBER working paper no. 14484.

ORourke, Kevin H., Alan M. Taylor and J. G. Williamson. 1996. "Factor Price Convergence in the Late Nineteenth Century," *International Economic Review* 37: 499-530.

O'Rourke, Kevin H. and Jeffrey G. Williamson. 1994. "Late 19th Century Anglo-American Factor Price Convergence: Were Heckscher and Ohlin Right?" *Journal of Economic History* 54: 892-916.

O'Rourke, Kevin H. and Jeffrey G. Williamson. 1999. *Globalization and History*. Cambridge, MA.: MIT Press.

Pomeranz, Kenneth. 2000. *The Great Divergence: China, Europe, and the Making of the Modern World Economy*. Princeton Economic History of the Western World series. Princeton and Oxford: Princeton University Press.

Rahman, Ahmed S. 2009. "The Road Not Taken - What Is The 'Appropriate' Path to Development When Growth Is Unbalanced," United States Naval Academy working paper no. 26.

Romer, Paul M. 1986. "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94: 1002-1037.

Romer, Paul M. 1990. "Endogenous Technological Change," *Journal of Political Economy* 98(5): S71-S102.

Segerstrom, Paul S., T. C. A. Anant, and Elias Dinopoulos. 1990. "A Schumpeterian Model of the Product Life Cycle," *American Economic Review*, 80: 1077-1092.

Temin, Peter. 1997. "Two Views of the British Industrial Revolution," *Journal of Economic History* 57(1): 63-82.

Young, Alwyn. 1998. "Growth without Scale Effects." *Journal of Political Economy*, 106: 41-63.

A Diversified Trade Equilibrium

With trade of goods y_1 and y_3 between the North and the South, productions in each region are given by (35) and (36).

For each region $c \in n, s$, the following conditions characterize the diversified trade equilibrium.

$$p_1^s = \frac{w_l^s}{A_1^s} \tag{37}$$

$$p_2^c = \left(\frac{1}{A_2^c}\right)(w_l^c)^{\gamma}(w_h^c)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{-\gamma} \tag{38}$$

$$p_3^n = \frac{w_h^n}{A_3^n} \tag{39}$$

$$\left(\frac{1}{A_1^c}\right)y_1^c + \left(\frac{1}{A_2^c}\right)(w_l^c)^{\gamma-1}(w_h^c)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}y_2^c = L^c \tag{40}$$

$$\left(\frac{1}{A_2^c}\right)(w_l^c)^{\gamma}(w_h^c)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}y_2^c + \left(\frac{1}{A_3^c}\right)y_3^c = H^c \tag{41}$$

$$y_1^n + a_1 Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^n)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{42}$$

$$y_1^s - Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^s)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{43}$$

$$y_2^c = \left(\frac{(1-\alpha)^{\sigma}(p_2^c)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^c)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^c)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^c)^{1-\sigma}}\right)\cdot Y^c \tag{44}$$

$$y_3^n - Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^n)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{45}$$

$$y_3^s + a_3 Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^s)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{46}$$

$$A_1^n\left(A_1^n L_1^n + a_1 Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right)A_2^{n\,\frac{\sigma-1}{\sigma}}\left(L^n - L_1^n\right)^{-\gamma-\sigma+\sigma\gamma}\left(H^n - H_3^n\right)^{\gamma+\sigma-\sigma\gamma-1} \tag{47}$$

$$A_3^n\left(A_3^n H_3^n - Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right)A_2^{n\,\frac{\sigma-1}{\sigma}}\left(L^n - L_1^n\right)^{-\gamma+\sigma\gamma}\left(H^n - H_3^n\right)^{\gamma-\sigma\gamma-1} \tag{48}$$

$$A_1^s \left(A_1^s L_1^s - Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right) A_2^{s\,\frac{\sigma-1}{\sigma}} \left(L^s - L_1^s\right)^{-\gamma-\sigma+\sigma\gamma} \left(H^s - H_3^s\right)^{\gamma+\sigma-\sigma\gamma-1} \tag{49}$$

$$A_3^s \left(A_3^s H_3^s + a_3 Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right) A_2^{s\,\frac{\sigma-1}{\sigma}} \left(L^s - L_1^s\right)^{-\gamma+\sigma\gamma} \left(H^s - H_3^s\right)^{\gamma-\sigma\gamma-1} \tag{50}$$

$$A_1^c = \left(N_{1,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{1,t}^c - N_{1,t-1}^c\right)\right) \left(\alpha p_1^c\right)^{\frac{\alpha}{1-\alpha}} \tag{51}$$

$$A_2^c = \left(N_{2,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{2,t}^c - N_{2,t-1}^c\right)\right) \left(\alpha p_2^c\right)^{\frac{\alpha}{1-\alpha}} \tag{52}$$

$$A_3^c = \left(N_{3,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{3,t}^c - N_{3,t-1}^c\right)\right) \left(\alpha p_3^c\right)^{\frac{\alpha}{1-\alpha}} \tag{53}$$

$$H^c = \left(\frac{\tau^{*c}}{b^c}\right) pop^c \tag{54}$$

$$L^c = \left(1 - \frac{\tau^{*c}}{b^c}\right) pop^c + n^c pop^c \tag{55}$$

$$n^c = \left(1 - \frac{\tau^{*c}}{b^c}\right) n_l^{*c} + \left(\frac{\tau^{*c}}{b^c}\right) n_h^{*c} \tag{56}$$

$$e^c = \frac{\tau^{*c}}{b^c} \tag{57}$$

$$\frac{p_1^n}{p_3^n} = \frac{Z_3}{aZ_1} \tag{58}$$

$$\frac{p_1^s}{p_3^s} = \frac{aZ_3}{Z_1} \tag{59}$$

Equations (37) - (39) are unit cost functions, (40) and (41) are full employment conditions, (42) - (46) denote regional goods clearance conditions, (47) - (50) equate the marginal products of raw factors, (51) - (53) describe sector-specific technologies, , (54) - (63) describe fertility, education and labor-types for each region, and (64) and (65) describe the balance of payments for each region. Solving this system for the unknowns p_1^n, p_1^s, p_2^n, p_2^s, p_3^n, p_3^s, y_1^n, y_1^s, y_2^n, y_2^s, y_3^n, y_3^s, w_l^n, w_l^s, w_h^n, w_h^s, L_1^n, L_1^s, H_3^n, H_3^s, A_1^n, A_2^n, A_3^n, A_1^s, A_2^s, A_3^s, L^n, L^s, H^n, H^s, n^n, n^s, e^n, e^s, Z_1 and Z_3 constitutes the static partial trade equilibrium.

Population growth for each region is given simply by

$$pop_t^c = n_{t-1}^c pop_{t-1}^c$$

Each region will produce all three goods so long as factors and technologies are "similar enough." If factors of production or technological levels sufficiently differ, the North produces only goods 2 and 3, while the South produces only goods 1 and 2. No other specialization scenario is possible for the following reasons: first, given that both the North and South have positive levels of L and H, full employment of resources implies that they cannot specialize completely in good 1 or good 3. Second, specialization solely in good 2 is not possible either, since a region with a comparative advantage in this good would also have a comparative advantage in either of the other goods. This implies that each country must produce at least two goods. Further, in such a scenario we cannot have one region producing goods 1 and 3: with different factor prices across regions, a region cannot have a comparative advantage in the production of both of these goods, regardless of the technological differences between the two regions. See Cunat and Maffezzoli (2002) for a fuller discussion.

B Specialized Trade Equilibrium

The specialized equilibrium is one where the North does not produce any good 1 and the South does not produce any good 3. Productions in each region are then given by

$$Y^n = \left(\frac{\alpha}{2}\left(aZ_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(y_2^n\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(y_3^n - Z_3\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{60}$$

$$Y^s = \left(\frac{\alpha}{2}\left(y_1^s - Z_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(y_2^s\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(aZ_3\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{61}$$

Once again, we do not permit any trade of good 2. For each region $c \in n, s$, the following conditions characterize this equilibrium.

$$p_1^s = \frac{w_l^s}{A_1^s} \tag{62}$$

$$p_2^c = \left(\frac{1}{A_2^c}\right)\left(w_l^c\right)^{\gamma}\left(w_h^c\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{-\gamma} \tag{63}$$

$$p_3^n = \frac{w_h^n}{A_3^n} \tag{64}$$

$$\left(\frac{1}{A_2^n}\right)\left(w_l^n\right)^{\gamma-1}\left(w_h^n\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}y_2^n = L^n \tag{65}$$

$$\left(\frac{1}{A_2^n}\right)\left(w_l^n\right)^{\gamma}\left(w_h^n\right)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}y_2^n + \left(\frac{1}{A_3^n}\right)y_3^n = H^n \tag{66}$$

$$\left(\frac{1}{A_1^s}\right)y_1^s + \left(\frac{1}{A_2^s}\right)\left(w_l^s\right)^{\gamma-1}\left(w_h^s\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}y_2^s = L^s \tag{67}$$

$$\left(\frac{1}{A_2^s}\right)(w_l^s)^{\gamma}(w_h^s)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}y_2^s = H^s \tag{68}$$

$$a_1Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^n)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{69}$$

$$y_1^s - Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^s)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{70}$$

$$y_2^c = \left(\frac{(1-\alpha)^{\sigma}(p_2^c)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^c)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^c)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^c)^{1-\sigma}}\right)\cdot Y^c \tag{71}$$

$$y_3^n - Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^n)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{72}$$

$$a_3Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^s)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{73}$$

$$A_3^n\left(A_3^nH_3^n - Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right)A_2^{n\,\frac{\sigma-1}{\sigma}}\left(L^n\right)^{-\gamma+\sigma\gamma}\left(H^n - H_3^n\right)^{\gamma-\sigma\gamma-1} \tag{74}$$

$$A_1^s\left(A_1^sL_1^s - Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right)A_2^{s\,\frac{\sigma-1}{\sigma}}\left(L^s - L_1^s\right)^{-\gamma-\sigma+\sigma\gamma}\left(H^s\right)^{\gamma+\sigma-\sigma\gamma-1} \tag{75}$$

$$A_1^s = \left(N_{1,t-1}^s + \alpha^{\frac{\alpha}{1-\alpha}}\left(N_{1,t}^s - N_{1,t-1}^s\right)\right)(\alpha p_1^s)^{\frac{\alpha}{1-\alpha}} \tag{76}$$

$$A_2^c = \left(N_{2,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}}\left(N_{2,t}^c - N_{2,t-1}^c\right)\right)(\alpha p_2^c)^{\frac{\alpha}{1-\alpha}} \tag{77}$$

$$A_3^n = \left(N_{3,t-1}^n + \alpha^{\frac{\alpha}{1-\alpha}}\left(N_{3,t}^n - N_{3,t-1}^n\right)\right)(\alpha p_n^c)^{\frac{\alpha}{1-\alpha}} \tag{78}$$

$$H^c = \left(\frac{\tau^{*c}}{b^c}\right)pop^c \tag{79}$$

$$L^c = \left(1 - \frac{\tau^{*c}}{b^c}\right)pop^c + n^c pop^c \tag{80}$$

$$n^c = \left(1 - \frac{\tau^{*c}}{b^c}\right)n_l^{*c} + \left(\frac{\tau^{*c}}{b^c}\right)n_h^{*c} \tag{81}$$

$$e^c = \frac{\tau^{*^c}}{b^c} \tag{82}$$

$$\frac{p_1^n}{p_3^n} = \frac{Z_3}{aZ_1} \tag{83}$$

$$\frac{p_1^s}{p_3^s} = \frac{aZ_3}{Z_1} \tag{84}$$