Improving Peak-to-Average power Ratio (PAR) and Probability of Error in OFDM-based WLAN

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Abstract – In this paper we investigate the use of complementary Golay sequences to achieve both a reduction of Peak-to-Average power Ratio (PAR) and a coding gain (reduction of Bit Error Probability) in OFDM systems. The achievement of these two simultaneous properties is of capital importance when using OFDM in hostile power-limited environments, such as the broadband WLAN that is being developed in WIND-FLEX project. After describing the main system parameters, BER performance curves are provided and some implementation details are discussed.

I. Introduction

Today's needs of bandwidth and flexibility are imposing the use of efficient modulations that may be fit to the characteristics of wireless channels. This is one of the reasons why multicarrier modulation techniques are finding growing interest for Wireless Local Area Networks (WLAN). Recent WLAN standards, such as Hiperlan type 2 [Etsi101] and IEEE 802.11a [Ieee802], have adopted OFDM for transmission of high bit rates in these networks.

WIND-FLEX Project (IST-1999-10025) was born with the purpose of designing and building a Wireless Indoor Flexible high bit-rate modem architecture offering up to 100 Mbps. The choice of OFDM (Orthogonal Frequency Division Multiplexing) is due to its good performance in multipath environments and the number of sub-carriers has been chosen so as to mitigate the indoor channel effects [Lob00]. Nevertheless, one of the main disadvantages of this modulation is its high PAR (Peak-to-Average power Ratio) requiring the use of linear HPAs (High Power Amplifiers) that are very power-inefficient and have an enormous impact on equipment's autonomy.

Thus, it is interesting to find techniques for the reduction of PAR in OFDM signals. There exist some of such techniques (Partial Transmit Sequence, Selective Mapping, ...) [Lcim99], but they normally have a negative impact in bandwidth efficiency, since they require the transmission of additional information. Also, the reduction achieved with those techniques is data-dependent, i. e. we obtain more or less PAR reduction depending on the characteristics of the input data sequence.

In this paper we analyse the use of complementary Golay sequences to achieve an improvement in the probability of error and a simultaneous data-independent reduction of PAR. This combined gain is of capital importance since we are aiming at high bit rates (in the order of 100 Mbps) in a hostile wireless environment.

The remainder of this paper is organised as follows. Complementary Golay sequences are introduced in section II for PAR reduction in OFDM. In this section, the system block diagram and the main parameters are described. In section III simulation results are provided showing the BER performance of these codes for different values of the main system parameters. Section IV deals with implementation details and we finally extract some conclusions in section V.

II. Complementary Golay sequences for PAR reduction in OFDM

The OFDM signal is basically an addition of modulations of N independent sub-carriers and only a few data sequences may cause all carriers to add in-phase and produce a signal with large PAR. We can try to avoid this type of data sequences.

Indeed, there exist certain types of codes that generate sequences that, once modulated, have their PAR confined into a certain margin. In the case of complementary Golay sequences, PAR is at most 3 dB independently of the number of sub- carriers that are being used. This is very interesting, given that the PAR of an OFDM signal is normally about $10 \cdot \log N\ (dB)$ in signals with a small number of sub-carriers and it is never lower than 13 dB in signals with a larger number of sub-carriers [Merch98].

In order to explore the PAR reduction and performance of complementary Golay sequences applied to OFDM, we introduce the coding to the data signal before Orthogonal Frequency Division Multiplexing as shown in the system block diagram of Figure 1.

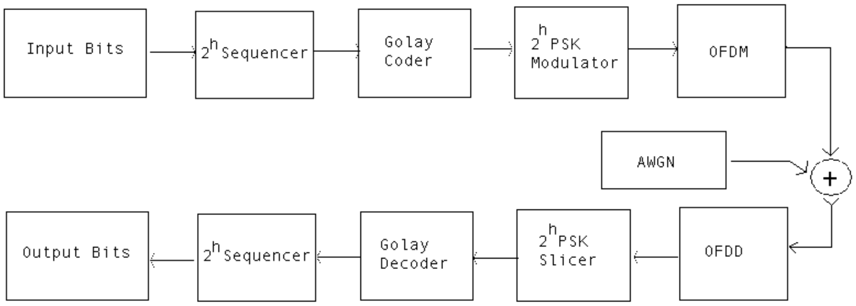

Figure 1. System block diagram

Before defining the parameters of complementary Golay sequences as they are applied to OFDM, we review the relationship of this kind of sequences with Reed-Muller codes, in order to be able to perform the coding and decoding operations in a straightforward way.

II.I Complementary Golay sequences and Reed-Muller codes

There exists a connection between Complementary Golay sequences, binary Reed–Muller codes (RM) and non-binary Reed–Muller codes (ZRM) [Davis97]: each of the *m!/2* cosets of $RM_{2^h}(1,m)$ in $ZRM_{2^h}(2,m)$ having a cosets representative of the form:

$$2^{h-1}\sum_{k=1}^{m-1} x_{\pi(k)} \cdot x_{\pi(k+1)} \qquad \text{[eq. 1]}$$

comprise one of $2^{h(m+1)}$ Golay sequences over $\mathbb{Z}_{2^h}$ of length 2^m, where $h > 1$, π is a permutation of $\{1...m\}$ and 2^m is the length of the complementary Golay sequence.

In addition, there is another interesting relationship: any sequence of the form

$$2^{h-1} \cdot \sum_{k=1}^{m-1} x_{\pi(k)} \cdot x_{\pi(k+1)} + \sum_{k=1}^{m} c_k \cdot x_k \qquad \text{[eq. 2]}$$

with $c_k \in \mathbb{Z}_{2^h}$ is a complementary Golay sequence with respect to each other.

If we take these two results, it is not difficult to develop a block algorithm for coding data sequences with the error correction capability of Reed–Muller codes (using first property) and good PAR characteristics of complementary Golay sequences (using second property).

II.II Main parameters of complementary Golay sequences

This Golay block coding algorithm has 3 design parameters that will be denoted by the letters *m*, *h* and *w*. First parameter is code length *m*, *h* is modulation order and *w* is number of base Golay sequences used.

We use first *w* bits to select the Golay base sequence in $ZRM_{2^h}(2,m)$, therefore $w \leq m!/2$; later we take next $m+1$ groups of *h* bits to build the final code combination, and finally, we map this sequence into a 2^h-PSK. This is the sequence that is introduced into the OFDM signal. In this way, we obtain a sequence that can be easily decoded in reception, with the error correction capabilities of block codes and with a PAR of 3 dB. Golay base sequences are constructed following [eq. 1].

It is important to note that when *w* is increased, the number of base codes increases too, and since in reception all base codes are used for decoding, complexity increases exponentially as *w* increase.

In reception, we use a variant of Reed–Muller decoding that also allows us to obtain the Golay base sequences being used. One of the principal characteristics of Reed–Muller codes is their ease of decoding with the Hadamard Transform. Moreover, this kind of algorithms are scalable, that is, with a decoder of length 2^n we can obtain another one that decodes length 2^{n+1} by just iterating once again. However, it should be pointed out that, for this reason, each time we increase the code length, the number of stages of the decoding algorithm is increased too.

We define the code rate (ratio of input-to-output-bits) for this scheme as:

$$R = \frac{w + (m+1)\cdot h}{2^m \cdot h} \qquad \text{[eq. 3]}$$

Figure 2 represents the value of the code rate (obtained form [eq. 3] with maximum *w* parameter for each *m*) for different code lengths (*m*) and modulation orders (*h*). We can see that the code rate decreases exponentially as the value of code length increases. Also, we can see that the modulation order does not affect very much the code rate, so we may select the value of this parameter with flexibility.

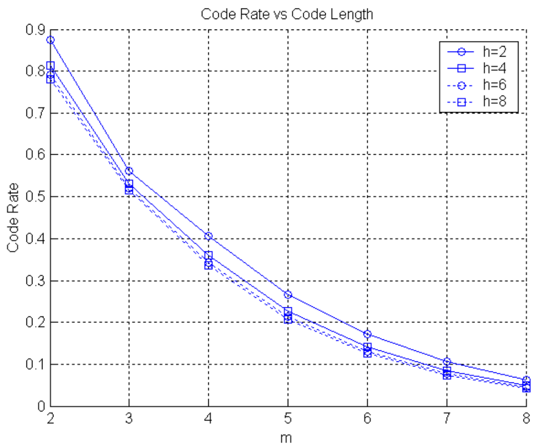

Figure 2. Code Rate vs. Code Length

III. Simulation Results

Both encoder and decoder have been implemented and their performance in the system block diagram of Figure 1 has been analysed via simulation in terms of error correction capabilities.

Error correction capabilities depend mainly on the code length because the larger this parameter, the more the redundancy, and thus the Hamming and Lee distances increase, leading us to an increase in the error correction capabilities. We can see this in Figure 3 in which we represent the Bit Error Rate (BER) for different values of E_b / N_0 and code length (*m*) when sub-carriers are 8-PSK-modulated. The number of sub-carriers in the OFDM signal is set to 2^m (i. e. the length of the complementary Golay sequence).

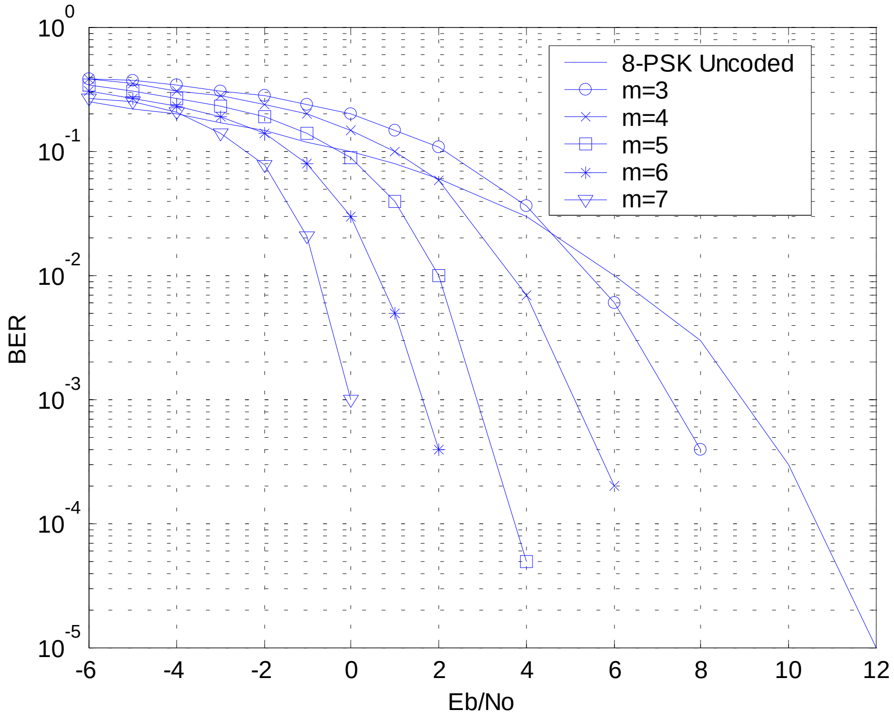

Figure 3. BER vs. E_b/N_0 for 8-PSK

In this figure we can see that as the code length increases, we have better error correction capabilities, but we can also see that for low E_b / N_0 we obtain worse results than those achieved without coding. This is because we are dealing with block codes, and when the code length is enough to correct the possible errors, the performance of the system is very good, but when the system fails, it fails in block and BER is increased.

We can also see that, when the code rate decreases, the performance is improved. This is due to the fact that performance depends mainly on *m* and *h*. As [eq. 3] shows, when *m* is increased, code rate decreases exponentially. Parameter *h* has minor influence. However, a smaller code rate does not always mean that performance is better. For example, the code rate for *w*=1, *m*=3, *h*=2 is 0.5625 and for *w*=1, *m*=3 and *h*=3 is 0.5417 and the performance in the first case is better than in the second. Thus it is only true that performance improves as code rate decreases if we modify just one parameter.

There is a trade-off between complexity (greater *m*) and coding gain. For 8-PSK modulation a 9 dB gain over uncoded 8-PSK-OFDM may be achieved at a BER= 10^{-3} when using *m*=7.

As regards to modulation order (*h*), BER increases for the same E_b / N_0 when *h* is increased. This means that the code length must be increased to maintain the BER, as can be seen in Figure 4 that shows BER vs. E_b / N_0, modulation order (QPSK and 8-PSK) and code length (*m*). Again, the number of sub-carriers in the OFDM signal is set to 2^m.

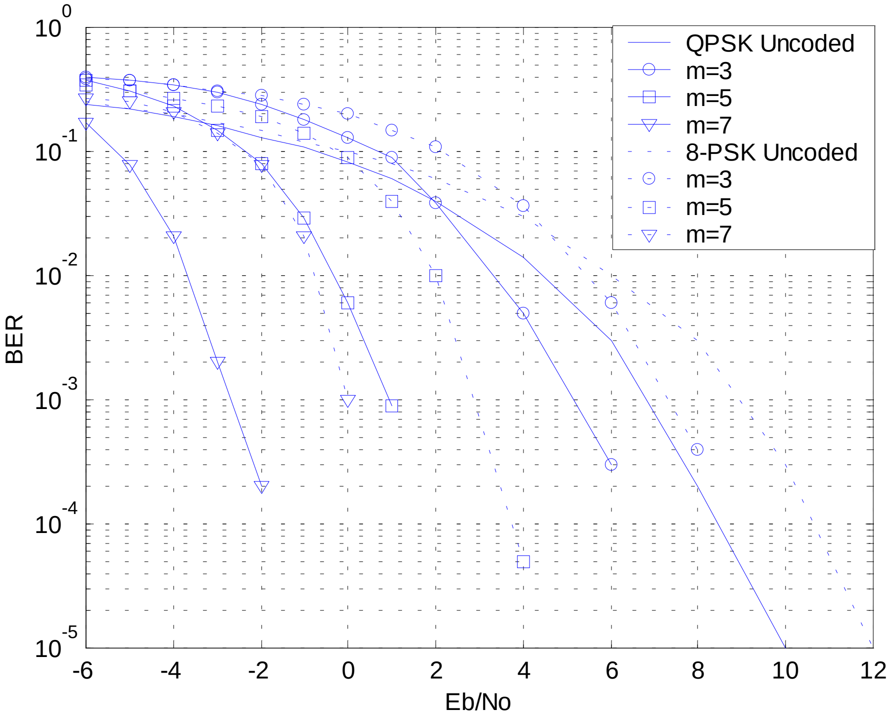

Figure 4. BER vs. E_b/N_0

In this figure we can see that when QPSK and 8-PSK modulations are used with codes of equal length, the error probability is several orders of magnitude lower for QPSK than for 8-PSK modulation (he coding gain for QPSK with respect to uncoded QPSK-OFDM is 10 dB at a BER= 10^{-3} when using *m*=7). This difference must be compensated with the code by increasing its length.

These results have been obtained using Montecarlo simulations for AWGN (Additive White Gaussian Noise) channels. In [Jones98] some results are given for Rayleigh channels that point to a satisfactory performance too. We aim at checking the code capabilities within the simulation environment that has been developed to analyse WIND-FLEX prototype [Lob00].

IV. Implementation

One disadvantage of the Golay encoder described in section II is the needed for a lookup table with all Golay base sequences, especially when *m* and *w* are large. This fact can make it prohibitive to implement the lookup table. However, we have found an algorithm to reduce the complexity required to build a specific Golay code. This algorithm uses the lexicographic order of permutation.

If we analyse the generation of all permutations in lexicographic order, we can see that they have a $(m-i)!$ relation, so in this way, we can generate one specific Golay base sequence at simulation runtime. Not all permutations are valid, only half of this m!. Symmetric permutations generate the same Golay code, but the first $\sum_{i=1}^{m-1}(m-i)!$ permutations are enough to obtain a good trade-off between the number of codes (w parameter) and complexity, since to determine which are the next permutations is not easy.

This reduction of complexity has no effect in the PAR reduction capabilities of the codes.

V. Conclusions

We have introduced Golay complementary sequences as a means to achieve both error correction and PAR reduction in OFDM signals. Simulation results have been provided that show the performance of these codes for a small number of sub-carriers, in the order of that selected for WIND-FLEX (256) and other OFDM-based WLANs (64 is a usual choice). The obtained results show that QPSK or 8-PSK modulations would be reasonable in WLAN environments (reduced number of sub-carriers) to obtain satisfactory results. Indeed coding gains up to 10 dB with a PAR reduced to 3 dB may be achieved.

Nevertheless, if the number of sub-carriers is increased like suggested for fourth generation environments (4G) [Ohmo00, Chuan00], we can use larger modulation orders since the code length may be larger too. However, as we increase the code length, the complexity of the system increases too, so a trade-off must be made between performance and complexity. In this sense, we have found an algorithm that simplifies implementation while maintaining a good performance.

Finally, we would like to point out that PAR in real signals is not equal to simulated PAR in sampled signals, so we are developing a demonstrator so as to be able to measure this characteristic.

References

[Etsi101] "*Broadband Radio Access Networks (BRAN); HIPERLAN Type 2; Physical (PHY) Layer*", ETSI TS 101 475 V1.1.1, April 2000.

[Ieee802] "*Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications: High-Speed Physical Layer in the 5 GHZ Band*", IEEE 802.11a-1999.

[Lob00] M. Lobeira, A. G. Armada, R. Torres, J. L. García, "Parameter estimation and indoor channel modelling at 17 GHz for OFDM-based broadband WLAN", Proceedings of IST Mobile Communications Summit, Galway, Ireland, 2000, pp: 29-33.

[Lcim99] L. Cimini Jr, N. R. Sollenberger. "Peak-to-Average Power Ratio Reduction of an OFDM Signal Using Partial Transmit Sequences", Proceedings of ICC, vol. 1, pp: 511-515, 1999.

[Merch98] S. Merchán, A. G. Armada, J. L. García, "OFDM Performance in Amplifier Nonlinearity", IEEE Transactions on Broadcasting, Vol. 44, no. 1, March 1998, pp: 106-114.

[Davis97] J. A. Davis and J. Jedwab, "Peak-to-Mean Power Control in OFDM, Golay Complementary sequences and Reed-Muller Codes", HP Laboratories Bristol, Dec. 1997.

[Jones98] A. E. Jones and A. Wilkinson, "Performance of Reed-Muller Codes with OFDM and a Maximum Likelihood Decoding Algorithm", HP Laboratories Bristol. Network Systems Department, Apr. 98.

[Ohmo00] S. Ohmori, Y. Yamao and N. Nakajima, "The future Generation of Mobile Communications based on Broadband Access Technologies", Communications Magazine, Dec. 2000.

[Chuan00] J. Chuang and N. Sollenberger, "Beyond 3G: Wideband Wireless Data Access Based on OFDM and Dynamic Packet Assignment", Communications Magazine, Jul. 2000.